Fixed-Time Synchronization of Coupled Memristive Complex-Valued Neural Networks*

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Abstract: This paper investigates the fixed-time synchronization of coupled memristive complex-valued neural networks (MCNNs) with time-varying delay. Firstly, a complex-valued sign function is introduced to avoid the common separation technique and a new fixed-time stability theorem is established to improve the existing ones. Afterward, by proposing a discontinuous control scheme and applying differential inclusion theory, some criteria are derived to achieve fixed-time synchronization of master-salve coupled MCNNs. The developed control scheme here simplifies the traditional design by excluding the linear part. Finally, a numerical example is given to validate of the theoretical results.

Key words: complex value; coupled neural network; fixed-time synchronization; memristor

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0 Introduction

Memristor was first proposed by Professor Chua in 1971[1] to describe the relationship between flux and charge. Since memristor can better simulate the mechanism of human brain memory and forgetting, many researchers have been motivated to introduce it into neural networks and a mass of memristive neural networks (MNNs) has been established. Nowadays, the study of dynamic behaviors of MNNs, such as stability[2] and synchronization [3], has attracted wide attention in various fields.

In recent years, complex-valued neurons have been successfully applied in pattern recognition and nonlinear filtering in view of their greater computing power and information storage capabilities compared with real-valued neurons. Currently, some interesting results about memristive complex-valued neural networks (MCNNs) have been reported [4,5]. In most existing theoretical results, complex-valued systems were separated into two real-valued systems and then the dynamic behaviors were investigated by using the common analytical technique for real-valued neural networks (RNNs). Although this method is effective, it also leads to complex calculation in theoretical analysis and the failure to reflect complex-valued features and superiorities. Therefore, it is of great significance to explore new methods to study MCNNs in theoretical analysis and practical applications.

Recently, fixed-time control has been vastly studied in various fields because of the faster convergence compared with traditional feedback control and the independence of convergence time on initial conditions in comparison with finite-time control. However, although the fixed-time synchronization of real-valued MNNs has been extensively studied [6,7], few scholars have explored the fixed-time synchronization problem of MCNNs because of the theoretical difficulty.

Inspired by the discussion above, the purpose of this paper is to investigate the fixed-time synchronization of coupled complex-valued memristive neural networks with time-varying delays. Firstly, to avoid separating the coupled MCNNs into two real-valued subsystems and designing two real-valued controllers in existing results [4,5], a complex-valued sign function and a new norm for complex-valued numbers are introduced. Secondly, a new fixed-time stability theorem is established

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to improve the existing result [8]. Based on the developed theorem, a new fixed-time controller is proposed for MCNNs to improve the previous control design [6,7] and some less conservative criteria are derived to realize fixed-time synchronization.

Notations: Let $\mathbb{C}^n$ be a space composed of all n-dimensional complex vectors and $\mathbb{R}^n$ be the space of n-dimensional real vectors. For any $z=\mathrm{Re}(z)+\mathbf{i}\mathrm{Im}(z)\in\mathbb{C}$, $\bar{z}$ is the conjugate of z, $\mathrm{Re}(z)$ is the real part of z, $\mathrm{Im}(z)$ is the imaginary part of z. In this paper, for any $z\in\mathbb{C}$, $|z|_1=|\mathrm{Re}(z)|+|\mathrm{Im}(z)|$. For any $Z=(z_1,z_2,\cdots,z_n)^T\in\mathbb{C}^n$, Z^H is the conjugate transposition of Z, $\|Z\|_1=\sum\limits_{k=1}^{n}|z_k|_1$. Define $[z]=\mathrm{sign}(\mathrm{Re}(z))+\mathbf{i}\mathrm{sign}(\mathrm{Im}(z))$ as the sign function for z and $[Z]=([z_1],[z_2],\cdots,[z_n])^T$ as the sign function of complex-valued vector Z. For any complex variable matrix $A(t)=(a_{ij}(t))_{N\times N}$, convex hull of A is $\overline{co}[A(t)]=(\overline{co}[a_{ij}])_{N\times N}$.

1 Preliminaries and model description

Referring the MCNN model in[9], consider a complex system coupled by N MCNNs, which can be described as follows

$$\dot{x}_i(t)=-D_i(x_i(t))x_i(t)+A_i(x_i(t))f(x_i(t))+B_i(x_i(t))f(x_i(t-\tau(t)))+I+c\sum_{j=1}^{N}\omega_{ij}g(x_j(t)),\quad i=1,2,\cdots,N, \tag{1}$$

where $x_i(t)=(x_{i1},x_{i2},\cdots,x_{in})^T\in\mathbb{C}^n$ is the state vector of the i th CMNN at time $t, 0\le\tau(t)\le\tau$, $f(x_i(t))$, and $f(x_i(t-\tau(t)))$ are n-dimensional vectors of continuous activation functions, I is an external input vector, $g(x_j(t))$ is an n-dimension coupling function vector associated with vector $x_j(t)$, $c\in\mathbb{R}$ is the coupling strength, $\Omega=(\omega_{ij})_{N\times N}\in\mathbb{R}^{N\times N}$ represents the coupling configuration matrix, in which $\omega_{ij}>0$ if there is a link between node i and j, otherwise, $\omega_{ij}=0$ and the diagonal elements satisfy $\omega_{ii}=-\sum\limits_{j=1,j\neq i}^{N}\omega_{ij}$. In addition, $D_i(x_i(t))=\mathrm{diag}(d_{i1}(x_{i1}(t)),\cdots,d_{in}(x_{in}(t)))$, $A_i(x_i(t))=(a^i_{kl}(x_{ik}(t)))_{n\times n}$, and $B_i(x_i(t))=(b^i_{kl}(x_{ik}(t)))_{n\times n}$ are complex-valued memristive link weight matrices, each component of them is defined as follows

$$d_{ik}(x_{ik}(t))=\begin{cases}\acute{d}_{ik}, & x_{ik}(t)\in\Pi_{ik},\\ \grave{d}_{ik}, & x_{ik}(t)\notin\Pi_{ik},\end{cases}\quad a^i_{kl}(x_{ik}(t))=\begin{cases}\acute{a}^i_{kl}, & x_{ik}(t)\in\Pi_{ik},\\ \grave{a}^i_{kl}, & x_{ik}(t)\notin\Pi_{ik},\end{cases}\quad b^i_{kl}(x_{ik}(t))=\begin{cases}\acute{b}^i_{kl}, & x_{ik}(t)\in\Pi_{ik},\\ \grave{b}^i_{kl}, & x_{ik}(t)\notin\Pi_{ik},\end{cases}$$

where $k=1,2,\cdots,n$, $\Pi_{ik}=\{\mathrm{Re}(x_{ik})+\mathbf{i}\mathrm{Im}(x_{ik})\in\mathbb{C};|\mathrm{Re}(x_{ik})|+|\mathrm{Im}(x_{ik})|\le Q_{ik}\}$, and $\acute{d}_{ik}\neq\grave{d}_{ik}$, $\acute{a}^i_{kl}\neq\grave{a}^i_{kl}$, $\acute{b}^i_{kl}\neq\grave{b}^i_{kl}$.

The initial values associated with system (1) are given as $x_i(s)=\varphi_i(s)$, where $s\in[-\tau,0]$, $\varphi_i(s)=(\varphi_{i1}(s),\varphi_{i2}(s),\cdots\varphi_{in}(s))^T\in\mathcal{I}$, and $\mathcal{I}=\mathcal{C}([-\tau,0],\mathbb{C}^n)$ denotes the Banach space of all continuous mapping from $[-\tau,0]$ to $\mathbb{C}^n$.

Obviously, the right side of system (1) is discontinuous because of switching meristive connection weights, then the solutions of system (1) considered in the following are intended in Filippov's sense. According to differential inclusion theory[10], it is not difficult to obtain the following set-valued mapping

$$\dot{x}_i(t)\in-\overline{co}[D_i(x_i(t))]x_i(t)+\overline{co}[A_i(x_i(t))]f(x_i(t))+I+\overline{co}[B_i(x_i(t))]f_i(x_i(t-\tau(t)))+c\sum_{j=1}^{N}\omega_{ij}g(x_j(t)),\quad i=1,2,\cdots,N \tag{2}$$

By using measurable selection theorem[11], there exist $\hat{d}_{ik}\in\overline{co}[d_{ik}(x_{ik}(t))]$, $\hat{a}^i_{kl}\in\overline{co}[a^i_{kl}(x_{ik}(t))]$ and $\hat{b}^i_{kl}\in\overline{co}[b^i_{kl}(x_{ik}(t))]$ such that for a.e. $t\ge0$,

$$\dot{x}_i(t)=-\hat{D}_ix_i(t)+\hat{A}_if(x_i(t))+\hat{B}_if(x_i(t-\tau(t)))+c\sum_{j=1}^{N}\omega_{ij}g(x_j(t))+I,\ i=1,2,\cdots,N \tag{3}$$

where $\hat{D}_i=\mathrm{diag}(\hat{d}_{i1},\hat{d}_{i2},\cdots,\hat{d}_{in})$, $\hat{A}_i=(\hat{a}^i_{kl})_{n\times n}$, and $\hat{B}_i=(\hat{b}^i_{kl})_{n\times n}$

To achieve the synchronization within fixed time, in this paper, we refer to system (1) as the master system, the slave system is given as follows

$$\dot{y}_i(t)=-D_i(y_i(t))y_i(t)+A_i(y_i(t))f(y_i(t))+B_i(y_i(t))f_i(y_i(t-\tau(t)))+c\sum_{j=1}^{N}\omega_{ij}g(y_j(t))+I+U_i(t),\quad i=1,2,\cdots,N \tag{4}$$

in which $y_i(t)=(y_{i1},y_{i2},\cdots,y_{in})^T\in\mathbb{C}^n$ represents the state vector of slave system (4), $D_i(y_i(t))$, $A_i(y_i(t))$ and $B_i(y_i(t))$ are memristor weights associated with complex state vector $y_i(t)$, whose definitions are similar to these in model (1), and $U_i(t)$

is a controller which will be designed later. For $i = 1,2,\cdots N$, the initial values of system (3) are given as $y_i(s) = \psi_i(s)$ for $s\in[-\tau,0]$, where $\psi_i(s)=(\psi_{i1}(s),\psi_{i2}(s),\cdots\psi_{in}(s))^T\in\mathcal{I}$.

Similarly, there exist $\check{d}_{ik}\in\overline{co}[d_{ik}(y_{ik}(t))]$, $\check{a}^i_{kl}\in\overline{co}[a_{kl}(y_{ik}(t))]$, and $\check{b}^i_{kl}\in\overline{co}[b_{kl}(y_{ik}(t))]$ such that for a.e. $t\geq 0$,

$$\dot{y}_i(t)=-\check{D}_iy_i(t)+\check{A}_if(y_i(t))+\check{B}_if(y_i(t-\tau(t)))+c\sum_{j=1}^{N}\omega_{ij}g(y_j(t))+I+U_i(t),\quad i=1,2,\cdots,N, \tag{5}$$

where $\check{D}_i=\mathrm{diag}(\check{d}_{i1},\check{d}_{i2},\cdots,\check{d}_{in})$, $\check{A}_i=(\check{a}^i_{kl})_{n\times n}$, and $\check{B}_i=(\check{b}^i_{kl})_{n\times n}$.

For any $1\leq i\leq N$, define the synchronization error as $e_i(t)=y_i(t)-x_i(t)$, from (3) and (5), the following error system can be obtained

$$\dot{e}_i(t)=-\check{D}_iy_i(t)+\hat{D}_ix_i(t)+\check{A}_if(y_i(t))-\hat{A}_if(x_i(t))+\check{B}_if(y_i(t-\tau(t)))-\hat{B}_if(x_i(t-\tau(t)))+c\sum_{j=1}^{N}\omega_{ij}\tilde{g}(e_j(t))+U_i(t), \tag{6}$$

where $\tilde{g}(e_j(t))=g(y_j(t))-g(x_j(t))$.

To obtain the main results, the following preliminaries are provided.

Assumption 1 For $k=1,2,\cdots,n$, there exist positive constants L_k, H_k, and ν_k such that for any $x_1,x_2\in\mathbb{C}$,

$$|f_k(x_1)-f_k(x_2)|_1\leq L_k|x_1-x_2|_1,\ |f_k(x_1)|_1\leq H_k,\ |g_k(x_1)-g_k(x_2)|_1\leq \nu_k|x_1-x_2|_1.$$

Definition 1 The master-slave systems (1) and (4) are called to be fixed-time synchronized, if there exist a fixed constant $T_{\max}\geq 0$ and $T^*(\phi,\psi)\geq 0$ satisfying $T(\phi,\psi)\leq T_{\max}$ for any $\phi,\psi\in\mathbb{C}^{Nn}$, such that $\lim_{t\to T^*(\phi,\psi)}\|e(t)\|=0$ and $\|e(t)\|=0$ for all $t\geq T^*(\phi,\psi)$, where $e(t)=(e_1(t)^T,e_2(t)^T,\cdots,e_N(t)^T)^T$, ϕ and ψ are the initial values of systems (1) and (4).

Lemma 1[12] Assume that $b_i\geq 0$ for $1\leq i\leq N$, $0<\theta\leq 1$ and $\delta>1$, then

$$\sum_{i=1}^{N}b_i^{\theta}\geq\Big(\sum_{i=1}^{N}b_i\Big)^{\theta},\quad \sum_{i=1}^{N}b_i^{\delta}\geq N^{1-\delta}\Big(\sum_{i=1}^{N}b_i\Big)^{\delta}.$$

Lemma 2[12] For any $Z(t)=(z_1(t),z_2(t),\cdots,z_n(t))^T\in\mathbb{C}^n$, the following equations holds:

$$[Z(t)]^HZ(t)+Z(t)^H[Z(t)]=2\|Z(t)\|_1,\ D^+Z(t)^H[Z(t)]+[Z(t)]^HZ(t)=[Z(t)]^H\dot{Z}(t)+\dot{Z}^H(t)[Z(t)].$$

From the recent work [13], for $z(t)\in\mathbb{C}$, the convex hull of real-valued sign function can be given by

$$\overline{co}([z(t)])=\overline{co}[\mathrm{sign}(\mathrm{Re}(z(t)))]+\overline{co}[\mathrm{sign}(\mathrm{Im}(z(t)))]\mathbf{i}.$$

Similar to the proof of Lemma 4 in [12], the following result is derived.

Lemma 3 For any $Z(t)\in\mathbb{C}^n$, denote $\overline{co}([Z(t)])=(\overline{co}([z_1(t)]),\overline{co}([z_2(t)]),\cdots,\overline{co}([z_n(t)]))^T$ and $\eta(t)=(\eta_1(t),\eta_2(t),\cdots,\eta_n(t))^T$ with $\eta_k(t)\in\overline{co}[z_k(t)]$, the following statements are true:

$$[Z(t)]^H\eta(t)+\eta(t)^H[Z(t)]=2\big\|[Z(t)]\big\|_1,\ Z(t)^H\eta(t)+\eta(t)^HZ(t)=2\|Z(t)\|_1\geq 2\|Z(t)\|_2.$$

Lemma 4[12] For any $z\in\mathbb{C}$, the following inequality holds:

$$z+\bar{z}=2\mathrm{Re}(z)\leq 2|z|_2\leq 2|z|_1.$$

Definition 2[14] Let $V(x(t)):\mathbb{R}^n\to\mathbb{R}$ be a regular, positive definite, radially unbounded function and $x(t)$ be absolutely continuous on any compact subinterval of $[0,+\infty)$, then $x(t)$ and $V(x(t))$ are differential for almost all $t\in[0,+\infty)$ and

$$\frac{\mathrm{d}V(x(t))}{\mathrm{d}t}=\xi^T\dot{x}(t),\quad\text{for all }\xi\in\partial V(x(t)),$$

where $\partial V(x(t))$ is the generalized gradient of $V(x(t))$.

Lemma 5 Assume that $V(x(t)) : \mathbb{R}^n \to \mathbb{R}^+$ is a regular, positive definite, radially unbounded function and $x(t)$ is absolutely continuous on any compact subinterval of $[0,+\infty)$, if

$$\dot{V}(x(t)) \le aV(x(t)) - bV^{\beta}(x(t)) - cV^{\theta}(x(t)),\ x(t) \in \mathbb{R}^n \setminus \{0\},$$

where $a \in \mathbb{R}$, $0 \le \beta < 1$, $\theta > 1$, and b, c are positive constants satisfying $\min\{b,c\} > a$, then $V(x(t)) = 0$ and $x(t) = 0$ for any $t \ge T$, and T is estimated by $T \le \frac{1}{a(1-\beta)} \ln(\frac{b}{b-a}) + \frac{1}{(c-a)(\theta-1)}$.

Proof Let $v(t) = V(x(t))$, it has

$$\dot{v}(t) \le av(t) - bv^{\beta}(t) - cv^{\theta}(t) \tag{7}$$

Since $0 \le \beta < 1, \theta > 1$ and $\min\{b,c\} > a$, then $\dot{v}(t) \le av(t) - bv^{\beta}(t) - cv^{\theta}(t) = -\Upsilon(v) \le 0$.

On the other hand, from (7), it has

$$T = \int_0^{v_0} \frac{1}{\Upsilon(v)}\,\mathrm{d}v = \frac{1}{1-\beta}\int_0^{v_0^{1-\beta}} \frac{\mathrm{d}v^{1-\beta}}{-av^{1-\beta} + b + cv^{\theta-\beta}},$$

where $v_0 = v(0)$. Let $\Psi = v^{1-\beta}$ and $\Psi_0 = v_0^{1-\beta}$, if $0 < \Psi_0 \le 1$,

$$T \le \frac{1}{1-\beta}\int_0^1 \frac{\mathrm{d}\Psi}{-a\Psi + b} = \frac{1}{a(1-\beta)} \ln(\frac{b}{b-a}).$$

If $\Psi_0 > 1$,

$$T = \frac{1}{1-\beta}\Big(\int_0^1 \frac{\mathrm{d}\Psi}{-a\Psi + b + c\Psi^{\frac{\theta-\beta}{1-\beta}}} + \int_1^{\Psi_0} \frac{\mathrm{d}\Psi}{-a\Psi + b + c\Psi^{\frac{\theta-\beta}{1-\beta}}}\Big) \le \frac{1}{a(1-\beta)} \ln(\frac{b}{b-a}) + \frac{1}{c-a}\frac{1}{\theta-1}.$$

Combining these two cases, one has

$$\int_0^{v_0} \frac{1}{\Upsilon(v)}\,\mathrm{d}v = T \le \frac{1}{a(1-\beta)} \ln(\frac{b}{b-a}) + \frac{1}{c-a}\frac{1}{\theta-1} < +\infty.$$

According to the finite-time stability theory in[14], $v(t) = 0, x(t) = 0$ for any $t \ge T$, and T does not depend on the initial value.

2 Main Results

In this section, we consider fixed-time synchronization between system (1) and system (4) by designing a discontinuous controller.

For any $k = 1,2,\cdots,n$, $i = 1,2,\cdots,N$, we introduce the following notations.

$$\tilde{a}^i_{kl} = \max\{|\acute{a}^i_{kl}|_1, |\grave{a}^i_{kl}|_1\}, \bar{a}^i_{kl} = |\grave{a}^i_{kl} - \acute{a}^i_{kl}|_1, \bar{b}^i_{kl} = \max\{|\acute{b}^i_{kl}|_1, |\grave{b}^i_{kl}|_1\},$$

$$\tilde{a}_{ik} = \sum_{l=1}^{n} \tilde{a}^i_{lk} L_k,\ \ \bar{a}_{ik} = \sum_{l=1}^{n} \bar{a}^i_{kl} H_l,\ \bar{b}_{ik} = 2\sum_{l=1}^{n} \bar{b}^i_{kl} H_l,$$

$$\tilde{d}_{ik} = \max\{|\mathrm{Im}(\acute{d}_{ik})| - \mathrm{Re}(\acute{d}_{ik}), |\mathrm{Im}(\grave{d}_{ik})| - \mathrm{Re}(\grave{d}_{ik})\},\quad \bar{d}_{ik} = |\grave{d}_{ik} - \acute{d}_{ik}|_1 Q_{ik}.$$

$$M = \mathrm{diag}\{m_1, m_2, \cdots, m_N\},\ \ \tilde{D}_k = \mathrm{diag}\{\tilde{d}_{1k}, \tilde{d}_{2k}, \cdots, \tilde{d}_{Nk}\},$$

$$\bar{D}_k = \mathrm{diag}\{\bar{d}_{1k}, \bar{d}_{2k}, \cdots, \bar{d}_{Nk}\},\ \tilde{A}_k = \mathrm{diag}\{\tilde{a}_{1k}, \tilde{a}_{2k}, \cdots, \tilde{a}_{Nk}\},$$

$$\bar{A}_k = \mathrm{diag}\{\bar{a}_{1k}, \bar{a}_{2k}, \cdots, \bar{a}_{Nk}\},\ \bar{B}_k = \mathrm{diag}\{\bar{b}_{1k}, \bar{b}_{2k}, \cdots, \bar{b}_{Nk}\},$$

$$\bar{e}_k(t) = (|e_{1k}(t)|_1, |e_{2k}(t)|_1, \cdots, |e_{Nk}(t)|_1)^T,\ \bar{e}^*_k(t) = \big(\big|[e_{1k}(t)]\big|_1, \big|[e_{2k}(t)]\big|_1, \cdots, \big|[e_{Nk}(t)]\big|_1\big)^T.$$

Theorem 1 Under Assumption 1, the controller in system (4) is designed by

$$U_i(t) = -[e_i(t)](m_i + \rho_1\|e_i(t)\|_1^{\beta} + \rho_2\|e_i(t)\|_1^{\theta}) \tag{8}$$

where $m_i \in \mathbb{R}$, $\rho_1, \rho_2 > 0$, $0 \le \beta < 1$, and $\theta > 1$, if

$$\min\{\rho_1, N^{1-\theta}\rho_2\} > \vartheta_1, \quad \bar{D}_k + \bar{A}_k + \bar{B}_k - M \le 0, \quad k = 1, 2, \cdots, n \tag{9}$$

where $\vartheta_1 = \max_{1\le k\le n}\{\lambda_{\max}(\tilde{D}_k + \tilde{A}_k + c\nu_k\Omega)\}$, then system (1) and system (4) are fixed-time synchronized. Moreover, the settling time T_0 is estimated by

$$T_0 \le T_0^* = \frac{1}{(1-\beta)\vartheta_1}\ln\frac{\rho_1}{\rho_1 - \vartheta_1} + \frac{1}{\theta - 1}\frac{1}{N^{1-\theta}\rho_2 - \vartheta_1}.$$

Proof By the measurable selection theorem[11], there exists a vector $\eta_i(t) = (\eta_{i1}(t), \cdots, \eta_{in}(t))^T$ with $\eta_{ik}(t) \in \overline{co}([e_{ik}(t)])$, such that for *a.e.* $t \ge 0$, $i = 1, 2 \cdots, N$,

$$\begin{aligned}\dot{e}_i(t) = &- \check{D}_i y_i(t) + \hat{D}_i(t) x_i(t) + \check{A}_i f(y_i(t)) - \hat{A}_i f(x_i(t)) + \check{B}_i f(y_i(t - \tau(t)))\\ &- \hat{B}_i f(x_i(t - \tau(t))) + c\sum_{i=1}^{N}\omega_{ij}\tilde{g}(e_j(t)) - \eta_i(t)(m_i + \rho_1\|e_i(t)\|_1^{\beta} + \rho_2\|e_i(t)\|_1^{\theta}).\end{aligned} \tag{10}$$

Consider the following Lyapunov function

$$V_1(e(t)) = \sum_{i=1}^{N}\|e_i(t)\|_1 = \frac{1}{2}\sum_{i=1}^{N}([e_i(t)]^H e_i(t) + e_i(t)^H [e_i(t)]),$$

where $e(t) = (\|e_1(t)\|_1, \|e_2(t)\|_1, \cdots, \|e_N(t)\|_1)^T$. For $e(t) \in \mathbb{R}^N \backslash \{0\}$, calculating the time derivative of $V_1(e(t))$ along the trajectories of the error system (10), one has

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}V_1(e(t)) &= \frac{1}{2}\sum_{i=1}^{N}([e_i(t)]^H\dot{e}_i(t) + \dot{e}_i^H(t)[e_i(t)])\\ &= \frac{1}{2}\sum_{i=1}^{N}\Big([e_i(t)]^H(-\check{D}_i y_i(t) + \hat{D}_i x_i(t)) + (-\check{D}_i y_i(t) + \hat{D}_i x_i(t))^H[e_i(t)]\Big)\\ &\quad + \frac{1}{2}\sum_{i=1}^{N}\Big([e_i(t)]^H(\check{A}_i f(y_i(t)) - \hat{A}_i f(x_i(t))) + (\check{A}_i f(y_i(t)) - \hat{A}_i f(x_i(t)))^H[e_i(t)]\Big)\\ &\quad + \frac{1}{2}\sum_{i=1}^{N}[e_i(t)]^H(\check{B}_i f(y_i(t - \tau(t))) - \hat{B}_i f(x_i(t - \tau(t))))\\ &\quad + \frac{1}{2}\sum_{i=1}^{N}(\check{B}_i f(y_i(t - \tau(t))) - \hat{B}_i f(x_i(t - \tau(t))))^H[e_i(t)]\\ &\quad + \frac{c}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}([e_i(t)]^H\omega_{ij}\tilde{g}(e_j(t)) + \tilde{g}^H(e_j(t))\omega_{ij}[e_i(t)])\\ &\quad - \frac{1}{2}\sum_{i=1}^{N}([e_i(t)]^H\eta_i(t) + \eta_i^H(t)[e_i(t)])(m_i + \rho_1\|e_i(t)\|_1^{\beta} + \rho_2\|e_i(t)\|_1^{\theta}).\end{aligned} \tag{11}$$

From the definition memristive link weights and classification discussion, it has

$$\frac{1}{2}\sum_{i=1}^{N}\Big([e_i(t)]^H(-\check{D}_i y_i(t) + \hat{D}_i x_i(t)) + (-\check{D}_i y_i(t) + \hat{D}_i x_i(t))^H[e_i(t)]\Big) \le \sum_{i=1}^{N}\sum_{k=1}^{n}\Big(\tilde{d}_{ik}\big|e_{ik}(t)\big|_1 + \bar{d}_{ik}\big|[e_{ik}(t)]\big|_1\Big). \tag{12}$$

Similarly, under Assumption 1, it has

$$\begin{aligned}&\frac{1}{2}\sum_{i=1}^{N}([e_i(t)]^H[\check{A}_i f(y_i(t)) - \hat{A}_i f(x_i(t))] + [\check{A}_i f(y_i(t)) - \hat{A}_i f(x_i(t))]^H[e_i(t)])\\ &\le \sum_{i=1}^{N}\sum_{k=1}^{n}\sum_{l=1}^{n}(\tilde{a}_{kl}^i L_l\big|e_{il}(t)\big|_1 + \bar{a}_{kl}^i H_l\big|[e_{ik}(t)]\big|_1) \le \sum_{i=1}^{N}\sum_{k=1}^{n}\Big(\tilde{a}_{ik}|e_{ik}(t)|_1 + \bar{a}_{ik}|[e_{ik}(t)]|_1\Big)\end{aligned} \tag{13}$$

$$\frac{1}{2}\sum_{i=1}^{N}[e_i(t)]^H(\check{B}_i f(y_i(t-\tau(t)))-\hat{B}_i f(x_i(t-\tau(t))))+\frac{1}{2}\sum_{i=1}^{N}(\check{B}_i f(y_i(t-\tau(t)))-\hat{B}_i f(x_i(t-\tau(t))))^H[e_i(t)]$$
$$\leq 2\sum_{i=1}^{N}\sum_{k=1}^{n}\sum_{l=1}^{n}\bar{b}^i_{kl}H_l\big|[e_{ik}(t)]\big|_1 \leq \sum_{i=1}^{N}\sum_{k=1}^{n}\bar{b}_{ik}|[e_{ik}(t)]|_1. \quad (14)$$

By using Assumption 1,

$$\frac{c}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}([e_i(t)]^H\omega_{ij}\tilde{g}(e_j(t))+\tilde{g}^H(e_j(t))\omega_{ij}[e_i(t)])=c\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{n}\omega_{ij}\mathrm{Re}(\overline{[e_{ik}(t)]}g_k(e_{jk}(t)))\leq c\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{n}\omega_{ij}\nu_k\big|e_{jk}(t)\big|_1. \quad (15)$$

According to Lemma 3, one can get

$$-\frac{1}{2}\sum_{i=1}^{N}([e_i(t)]^H\eta_i(t)+\eta_i^H(t)[e_i(t)])(m_i+\rho_1\|e_i(t)\|_1^{\beta}+\rho_2\|e_i(t)\|_1^{\theta})$$
$$\leq -\sum_{i=1}^{N}\sum_{k=1}^{n}\big|[e_{ik}(t)]\big|_1 m_i-\sum_{i=1}^{N}\big\|[e_i(t)]\big\|_1(\rho_1\|e_i(t)\|_1^{\beta}+\rho_2\|e_i(t)\|_1^{\theta})$$
$$\leq -\sum_{i=1}^{N}\sum_{k=1}^{n}\big|[e_{ik}(t)]\big|_1 m_i-\sum_{i=1}^{N}(\rho_1\|e_i(t)\|_1^{\beta}+\rho_2\|e_i(t)\|_1^{\theta}). \quad (16)$$

Substituting (12)-(16) into (11), from Lemma 1 and condition (9), we can obtain that

$$\frac{\mathrm{d}}{\mathrm{d}t}V_1(e(t))\leq\sum_{k=1}^{n}I_N^T(\tilde{D}_k+\tilde{A}_k+c\nu_k\Omega)\bar{e}_k(t)+\sum_{k=1}^{n}I_N^T(\bar{D}_k+\bar{A}_k+\bar{B}_k-M)\bar{e}_k^*(t)-\sum_{i=1}^{N}(\rho_1\|e_i(t)\|_1^{\beta}+\rho_2\|e_i(t)\|_1^{\theta})$$
$$\leq\vartheta_1 V(e(t))-\rho_1V^{\beta}(e(t))-\rho_2N^{1-\theta}V^{\theta}(e(t)). \quad (17)$$

By (17) and Lemma 5, system (1) and system (4) can be synchronized under the control scheme (8) in a fixed time T_0 which is estimated by

$$T_0\leq T_0^*=\frac{1}{(1-\beta)\vartheta_1}\ln\frac{\rho_1}{\rho_1-\vartheta_1}+\frac{1}{\theta-1}\frac{1}{N^{1-\theta}\rho_2-\vartheta_1}.$$

This completes the proof of Theorem 1.

When the nonlinear is reduced to linear case, that is, $g(v)=\Gamma v$ for $v\in\mathbb{C}^n$ and $\Gamma=\mathrm{diag}\{\gamma_1,\gamma_2,\cdots,\gamma_n\}$ with $\gamma_k\in\mathrm{R}$ in systems (1) and (4), the following result is easily obtained.

Corollary 1 Under Assumption 1 and the control law (8), if

$$\min\{\rho_1,\ N^{1-\alpha}\rho_2\}>\vartheta_1^*,\ \bar{D}_k+\bar{A}_k+\bar{B}_k-M\leq 0,\ \ k=1,2,\cdots,n,$$

where $\vartheta_1^*=\max_{1\leq k\leq n}\{\lambda_{max}(\tilde{D}_k+\tilde{A}_k+c\gamma_k\Omega)\}$, then system (1) and system (4) reach synchronization in a fixed time which is estimated by

$$T_0\leq T_1^*=\frac{1}{(1-\beta)\vartheta_1^*}\ln\frac{\rho_1}{\rho_1-\vartheta_1^*}+\frac{1}{\theta-1}\frac{1}{N^{1-\theta}\rho_2-\vartheta_1^*}.$$

Remark 1 In Theorem 1, if $\beta=0$, the estimate of T_0 is reduced to the following form

$$T_0\leq T_2^*=\frac{1}{\vartheta_1}\ln\frac{\rho_1}{\rho_1-\vartheta_1}+\frac{1}{\theta-1}\frac{1}{N^{1-\theta}\rho_2-\vartheta_1}.$$

Remark 2 In the previous results[4,5] on dynamic behaviors of MCNNs, the separation method was utilized to obtain two real-valued subsystems and the main results were derived by analyzing these reduced real-valued models. Different from the separation technique, the complex-valued sign function is introduced in this paper to directly design the complex-valued control scheme and construct Lyapunov function, and the theoretical analysis is finished in the complex field.

Remark 3 In the most of results on fixed-time synchronization[6,7], the linear term about the errors is indispensable in the design of controllers. Unlike the common design, the control law given in this paper is simplified by excluding the linear part. In addition, Lemma 5 generalized the existing fixed-time stability into discontinuous system and improved the estimation of the settling time provided in [8].

3 Numerical simulations

In this section, a numerical example is given to verify the theoretical results.

In systems (1) and (4), choose $i = 1, 2, \cdots, 12$, $n = 2$, $\Pi_{ik} = \{x_{ik}(t) \in \mathbb{C};\ |\mathrm{Re}(x_{ik}(t))| + |\mathrm{Im}(x_{ik}(t))| \leq 1.5,\ k = 1, 2\}$, $f_k(x_{ik}(t)) = \tanh(\mathrm{Re}(x_{ik}(t))) + \mathbf{i}\sin(\mathrm{Im}(x_{ik}(t)))$ for $k = 1, 2$, memristive link weight is given as follows: when $x_{ik}(t), y_{ik}(t) \in \Pi_{ik}$, $d_{i1} = 1.07 + 0.22\mathbf{i}, d_{i2} = 0.49 - 0.60\mathbf{i}$, $a^i_{11} = 2.20 - 1.03\mathbf{i}, a^i_{12} = -0.14 + 0.60\mathbf{i}, a^i_{21} = -5.10 + 3.49\mathbf{i}, a^i_{22} = 3.00 - 2.02\mathbf{i}$ and $b^i_{11} = -1.59 + 0.10\mathbf{i}, b^i_{12} = -1.19 - 0.13\mathbf{i}, b^i_{21} = -0.10 - 0.42\mathbf{i}, b^i_{22} = -2.10 - 1.90\mathbf{i}$, and when $x_{ik}(t), y_{ik}(t) \notin \Pi_{ik}$, $d_{i1} = 0.98 + 0.12\mathbf{i}, d_{i2} = 0.51 - 0.62\mathbf{i}$, $a^i_{11} = 2.10 - 0.99\mathbf{i}, a^i_{12} = -0.11 + 0.56\mathbf{i}, a^i_{21} = -5.12 + 3.51\mathbf{i}, a^i_{22} = 3.10 - 2.08\mathbf{i}$ and $b^i_{11} = -1.65 + 0.15\mathbf{i}, b^i_{12} = -1.11 - 0.12\mathbf{i}, b^i_{21} = -0.19 - 0.40\mathbf{i}, b^i_{22} = -1.99 - 1.89\mathbf{i}$.

Under those parameters, the chaotic attractors of the real part and the imaginary part for $x_i(t)$ in system (1) without the coupling are shown in Fig 1 and Fig 2, in which $x_1(0) = -2.18 - 1.09\mathbf{i}$ and $x_2(0) = -1.97 + 1.22\mathbf{i}$.

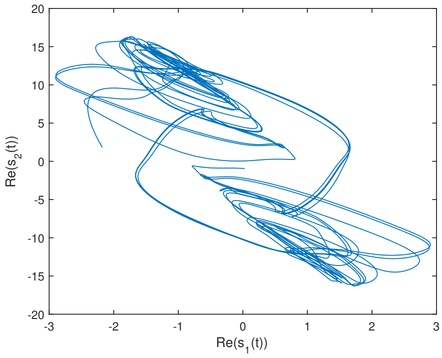

Fig 1 Phase trajectory of real part

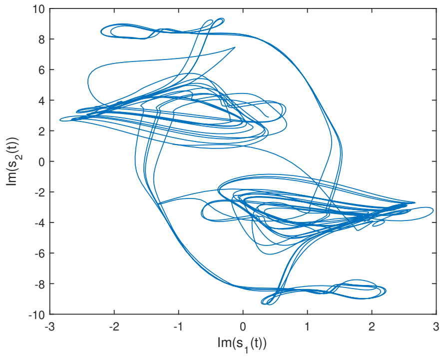

Fig 2 Phase trajectory of imaginary part

For any $1 \leq i \leq 12$, $k = 1, 2$, choose coupling function $g_k(x_{ik}(t)) = \tanh(\mathrm{Re}(x_{ik}(t))) + \mathbf{i}\tanh(\mathrm{Im}(x_{ik}(t)))$. By simple computation, $L_k = \nu_k = 1$, and $H_k = 2$. In control scheme (8), select control parameters $m_i = 14.70$, $\rho_1 = 11.20$, $\rho_2 = 34.26$, and $\theta = 1.42$. From Theorem 1, systems (1) and (4) are fixed-time synchronized, which is shown in Fig 3 and Fig 4 when $\beta = 0.5$ and $\beta = 0$ respectively, and the settling time can be given as $T^*_0 = 22.00$, $T^*_2 = 21.82$.

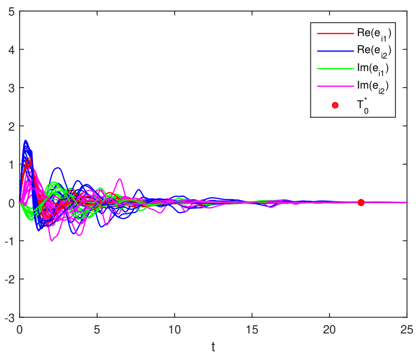

Fig 3 Synchronized error for $\beta = 0.5$

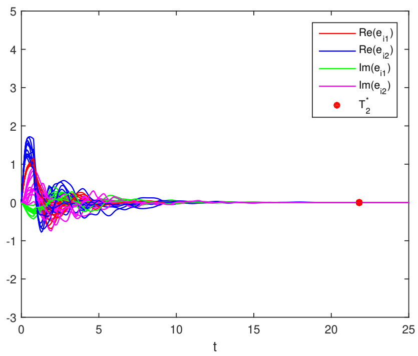

Fig 4 Synchronized error for $\beta = 0$

4 Conclusion

In this paper, the fixed-time synchronization has been discussed for a class of coupled MCNNs with time-varying delay. By introducing the complex-valued sign function and establishing a new fixed-time stability theorem, a discontinuous complex-valued controller has been designed and a new Lyapunov function has been proposed in theoretical analysis. The proposed control scheme simplified the traditional design by excluding the linear part. Further work will focus on the fixed-time control of impulsive systems and spatiotemporal models.

耦合忆阻复值神经网络的固定时间同步*

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摘 要：本文研究了具有时变时滞的耦合忆阻复值神经网络的固定时间同步. 首先，引入了复值符号函数来避开传统的分离方法，并建立了一个新的固定时间稳定定理来改进已有的相关结论；然后，通过设计不连续控制策略，利用微分包含理论，建立了主从耦合忆阻复值神经网络的固定时间同步判据. 本文的控制策略并不包含线性部分，简化了传统的控制设计；最后，通过数值实例对理论结果进行了验证.

关键词：复值；耦合神经网络；固定时间同步；忆阻

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0 引 言

1971年，蔡少棠教授首次提出了忆阻器, 并以此刻画电荷和磁通量间的关系[1]. 由于忆阻器能够模拟人脑的记忆和遗忘机制, 因而被研究者引入到神经网络, 建立了忆阻神经网络(MNNs). 目前, 诸如稳定性[2]、同步[3]等 MNNs 的动力学行为已被各领域学者广泛研究. 另一方面, 由于比实值神经元具有更高的计算能力和信息存储能力, 近年来复值神经元在模式识别、非线性滤波等方面得到了广泛的应用, 并且忆阻复值神经网络(MCNNs)也成为了当前的研究热点[4,5]. 而在大多数的研究中, 作者先将复值系统分离成两个实值子系统, 再利用实值神经网络的经典理论分析两个实值系统的动力学行为. 虽然这种分离方法是有效的, 但忽略了网络的复值特征和优势, 也会给理论分析和计算带来一定的困难. 因此, 探索分析 MCNNs 的新方法具有重要的理论价值和实际意义.

固定时间控制比传统的反馈控制具有更快的收敛速度, 相较于有限时间控制, 停息时间又不受初始条件的限制, 因而近年来被各领域学者广泛研究. 尽管目前已有相当丰富的实值 MNNs 的固定时间同步结果[6,7], 而忆阻复值神经网络却由于理论分析上的困难极少受到关注.

基于上述分析, 本文将研究具有时变时滞的耦合复值忆阻神经网络的固定时间同步. 首先, 为了避免现有结果[4,5]中将耦合忆阻复值神经网络分离为两个实值子系统, 再设计两个实值控制器的复杂分析方法, 本文引入了复值符号函数和复数域中的新范数, 使控制器设计、Lyapunov 函数的选取以及理论分析都能够在复数域内直接进行. 其次, 建立并证明了一个新的固定时间稳定性定理, 改进了现有的结果[8]. 基于该定理, 对耦合忆阻复值神经网络设计了新的固定时间控制策略, 简化了以往的控制器设计, 并得到了低保守性的固定时间同步准则.

记号: 在本文中, $\mathbb{C}^n$ 表示全体 n 维复向量构成的空间, $\mathbb{R}^n$ 代表全体 n 维实向量构成的空间. 对任意复数 $z=\mathrm{Re}(z)+\mathbf{i}\mathrm{Im}(z)$, $\bar{z}$ 表示 z 的共轭, $\mathrm{Re}(z)$ 是 z 的实部, $\mathrm{Im}(z)$ 是 z 的虚部. 对任意的复向量 $Z=(z_1,z_2,\cdots,z_n)^T\in\mathbb{C}^n$, Z^H 表示向量 Z 的共轭转置, $\|Z\|_1=\sum\limits_{k=1}^{n}|z_k|_1$. $[z]=\mathrm{sign}(\mathrm{Re}(z))+\mathbf{i}\mathrm{sign}(\mathrm{Im}(z))$ 表示复数 z 的符号函数, $[Z]=([z_1],[z_2],\cdots,[z_n])^T$ 表示复向量 Z 的符号函数. 此外对任意复变矩阵 $A(t)=(a_{ij}(t))_{N\times N}$, $\overline{co}[A(t)]=(\overline{co}[a_{ij}])_{N\times N}$ 代表矩阵 $A(t)$ 的凸包.

1 模型描述及预备知识

参考 Chen 等提出的 MCNN 模型[9], 本文考虑由 N 个 MCNNs 耦合而成的复杂网络, 其动力学模型描述为:

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$$\dot{x}_i(t)=-D_i(x_i(t))x_i(t)+A_i(x_i(t))f(x_i(t))+B_i(x_i(t))f(x_i(t-\tau(t)))+I+c\sum_{j=1}^{N}\omega_{ij}g(x_j(t)),\quad i=1,2,\cdots,N, \tag{1}$$

其中: $x_i(t)=(x_{i1},x_{i2},\cdots,x_{in})^T\in\mathbb{C}^n$ 是第 i 个 MCNN 在 t 时刻的状态向量, $0\le\tau(t)\le\tau$, $f(x_i(t))$ 和 $f(x_i(t-\tau(t)))$ 表示 n 维连续激活函数向量, I 表示网络的外部输入向量, $g(x_j(t))$ 是关于状态向量 $x_j(t)$ 的 n 维非线性耦合函数向量, $c\in\mathbb{R}$ 表示耦合权重, $\Omega=(\omega_{ij})_{N\times N}\in\mathbb{R}^{N\times N}$ 是网络的耦合矩阵, 这里如果节点 i 和 j 之间存在一条连边, 那么 $\omega_{ij}>0$, 否则 $\omega_{ij}=0$, 并且 Ω 的对角元满足: $\omega_{ii}=-\sum\limits_{j=1,j\neq i}^{N}\omega_{ij}$. 此外, $D_i(x_i(t))=\mathrm{diag}\big(d_{i1}(x_{i1}(t)),\cdots,d_{in}(x_{in}(t))\big)$, $A_i(x_i(t))=\big(a^i_{kl}(x_{ik}(t))\big)_{n\times n}$, $B_i(x_i(t))=\big(b^i_{kl}(x_{ik}(t))\big)_{n\times n}$ 表示忆阻连接权重矩阵, 其分量分别定义为

$$d_{ik}(x_{ik}(t))=\begin{cases}\acute{d}_{ik}, & x_{ik}(t)\in\Pi_{ik},\\ \grave{d}_{ik}, & x_{ik}(t)\notin\Pi_{ik},\end{cases}\quad a^i_{kl}(x_{ik}(t))=\begin{cases}\acute{a}^i_{kl}, & x_{ik}(t)\in\Pi_{ik},\\ \grave{a}^i_{kl}, & x_{ik}(t)\notin\Pi_{ik},\end{cases}$$

$$b^i_{kl}(x_{ik}(t))=\begin{cases}\acute{b}^i_{kl}, & x_{ik}(t)\in\Pi_{ik},\\ \grave{b}^i_{kl}, & x_{ik}(t)\notin\Pi_{ik},\end{cases}$$

其中: $k=1,2,\cdots,n$, $\Pi_{ik}=\{\mathrm{Re}(x_{ik})+\mathbf{i}\mathrm{Im}(x_{ik})\in\mathbb{C};|\mathrm{Re}(x_{ik})|+|\mathrm{Im}(x_{ik})|\le Q_{ik}\}$, 并且 $\acute{d}_{ik}\neq\grave{d}_{ik}$, $\acute{a}^i_{kl}\neq\grave{a}^i_{kl}$, $\acute{b}^i_{kl}\neq\grave{b}^i_{kl}$.

系统 (1) 的初始值为 $x_i(s)=\varphi_i(s)$, 这里 $s\in[-\tau,0]$, $\varphi_i(s)=(\varphi_{i1}(s),\varphi_{i2}(s),\cdots\varphi_{in}(s))^T\in\mathcal{I}$, $\mathcal{I}=\mathcal{C}([-\tau,0],\mathbb{C}^n)$ 表示将 $[-\tau,0]$ 映到 $\mathbb{C}^n$ 的全体连续函数构成的 Banach 空间.

由于忆阻连接权重具有切换性, 因此系统 (1) 的右端是不连续的. 鉴于此, 我们将考虑系统 (1) 在 Filippov 意义下的解. 利用微分包含理论[10], 不难得到如下集值映射:

$$\begin{aligned}\dot{x}_i(t)\in&-\overline{co}[D_i(x_i(t))]x_i(t)+\overline{co}[A_i(x_i(t))]f(x_i(t))+I\\&+\overline{co}[B_i(x_i(t))]f_i(x_i(t-\tau(t)))+c\sum_{j=1}^{N}\omega_{ij}g(x_j(t)),\quad i=1,2,\cdots,N.\end{aligned}\tag{2}$$

由可测选择理论[11], 存在 $\hat{d}_{ik}\in\overline{co}[d_{ik}(x_{ik}(t))]$, $\hat{a}^i_{kl}\in\overline{co}[a^i_{kl}(x_{ik}(t))]$ 和 $\hat{b}^i_{kl}\in\overline{co}[b^i_{kl}(x_{ik}(t))]$ 使得对几乎所有的 $t\ge0$,

$$\dot{x}_i(t)=-\hat{D}_ix_i(t)+\hat{A}_if(x_i(t))+\hat{B}_if(x_i(t-\tau(t)))+c\sum_{j=1}^{N}\omega_{ij}g(x_j(t))+I,\ i=1,2,\cdots,N, \tag{3}$$

其中: $\hat{D}_i=\mathrm{diag}\big(\hat{d}_{i1},\hat{d}_{i2},\cdots,\hat{d}_{in}\big)$, $\hat{A}_i=\big(\hat{a}^i_{kl}\big)_{n\times n}$, $\hat{B}_i=\big(\hat{b}^i_{kl}\big)_{n\times n}$.

为实现固定时间同步, 将系统 (1) 作为主系统, 对应的从系统为:

$$\begin{aligned}\dot{y}_i(t)=&-D_i(y_i(t))y_i(t)+A_i(y_i(t))f(y_i(t))+B_i(y_i(t))f_i(y_i(t-\tau(t)))\\&+c\sum_{j=1}^{N}\omega_{ij}g(y_j(t))+I+U_i(t),\quad i=1,2,\cdots,N,\end{aligned}\tag{4}$$

其中: $y_i(t)=(y_{i1},y_{i2},\cdots,y_{in})^T\in\mathbb{C}^n$ 代表从系统 (4) 的状态向量, $D_i(y_i(t))$, $A_i(y_i(t))$ 和 $B_i(y_i(t))$ $y_i(t)$ 是关于状态向量 $y_i(t)$ 的忆阻连接权矩阵, 其定义与模型 (1) 类似, $U_i(t)$ 表示将要设计的控制策略. 对任意的 $i=1,2,\cdots N$, 系统 (4) 的初值为 $y_i(s)=\psi_i(s)$, 这里 $s\in[-\tau,0]$, 且$\psi_i(s)=(\psi_{i1}(s),\psi_{i2}(s),\cdots\psi_{in}(s))^T\in\mathcal{I}$.

同理, 存在 $\check{d}_{ik}\in\overline{co}[d_{ik}(y_{ik}(t))]$, $\check{a}^i_{kl}\in\overline{co}[a_{kl}(y_{ik}(t))]$ 以及 $\check{b}^i_{kl}\in\overline{co}[b_{kl}(y_{ik}(t))]$ 使得对几乎所有的 $t\ge0$,

$$\dot{y}_i(t)=-\check{D}_iy_i(t)+\check{A}_if(y_i(t))+\check{B}_if(y_i(t-\tau(t)))+c\sum_{j=1}^{N}\omega_{ij}g(y_j(t))+I+U_i(t),\quad i=1,2,\cdots,N, \tag{5}$$

这里$\check{D}_i=\mathrm{diag}\big(\check{d}_{i1},\check{d}_{i2},\cdots,\check{d}_{in}\big)$, $\check{A}_i=\big(\check{a}^i_{kl}\big)_{n\times n}$, $\check{B}_i=\big(\check{b}^i_{kl}\big)_{n\times n}$.

对任意的 $1\le i\le N$, 定义同步误差为 $e_i(t)=y_i(t)-x_i(t)$, 那么由 (3) 和 (5) 可得,

$$\begin{aligned}\dot{e}_i(t)=&-\check{D}_iy_i(t)+\hat{D}_ix_i(t)+\check{A}_if(y_i(t))-\hat{A}_if(x_i(t))+\check{B}_if(y_i(t-\tau(t)))\\&-\hat{B}_if(x_i(t-\tau(t)))+c\sum_{j=1}^{N}\omega_{ij}\tilde{g}(e_j(t))+U_i(t),\end{aligned}\tag{6}$$

其中: $\tilde{g}(e_j(t))=g(y_j(t))-g(x_j(t))$.

为了得到本文的主要结论, 还需引入如下引理及假设.

假设 1 对任意的 $k=1,2,\cdots,n$, 存在正常数 L_k, H_k 以及 ν_k 使得对任意的 $x_1,x_2\in\mathbb{C}$,

$$|f_k(x_1)-f_k(x_2)|_1\le L_k|x_1-x_2|_1,\ |f_k(x_1)|_1\le H_k,\ |g_k(x_1)-g_k(x_2)|_1\le \nu_k|x_1-x_2|_1.$$

定义 1 若存在正常数 $T^*(\phi,\psi)$, 使得

$$\lim_{t\to T^*(\phi,\psi)}\|e(t)\|=0,\ \|e(t)\|=0,\ t\ge T^*(\phi,\psi),$$

并且存在正常数 $T_{\max}$, 使得对任意的 $\phi,\psi\in\mathbb{C}^{Nn}$ 有 $T^*(\phi,\psi)\le T_{\max}$, 则称主从耦合 MCNNs (1) 和 (4) 是固定时间同步的, 其中 $e(t)=(e_1(t)^T,e_2(t)^T,\cdots,e_N(t)^T)^T$, ϕ 和 ψ 分别为系统 (1) 和 (4) 的初始值.

引理 1[12] 对任意的 $i=1,2,\cdots,N$, 假设 $b_i\ge 0$, 并且 $0<\theta\le 1$, $\delta>1$, 那么

$$\sum_{i=1}^{N}b_i^{\theta}\ge\Big(\sum_{i=1}^{N}b_i\Big)^{\theta},\quad \sum_{i=1}^{N}b_i^{\delta}\ge N^{1-\delta}\Big(\sum_{i=1}^{N}b_i\Big)^{\delta}.$$

引理 2[12] 对任意复向量 $Z(t)=(z_1(t),z_2(t),\cdots,z_n(t))^T\in\mathbb{C}^n$, 下列性质成立:

$$[Z(t)]^HZ(t)+Z(t)^H[Z(t)]=2\|Z(t)\|_1,$$

$$D^+Z(t)^H[Z(t)]+[Z(t)]^HZ(t)=[Z(t)]^H\dot{Z}(t)+\dot{Z}^H(t)[Z(t)].$$

参考文献[13], 对任意的 $z(t)\in\mathbb{C}$, 复值符号函数的凸包为:

$$\overline{co}([z(t)])=\overline{co}[\mathrm{sign}(\mathrm{Re}(z(t)))]+\overline{co}[\mathrm{sign}(\mathrm{Im}(z(t)))]\mathbf{i}.$$

对任意复向量 $Z(t)\in\mathbb{C}^n$, 记 $\overline{co}([Z(t)])=\big(\overline{co}([z_1(t)]),\cdots,\overline{co}([z_n(t)])\big)^T$ 以及 $\eta(t)=(\eta_1(t),\cdots,\eta_n(t))^T$, 这里 $\eta_k(t)\in\overline{co}[z_k(t)]$. 类似于文献[12]中的引理 4, 不难得到以下结论.

引理 3 对任意复向量 $Z(t)\in\mathbb{C}^n$, 如下结论成立:

$$\begin{gathered}[Z(t)]^H\eta(t)+\eta(t)^H[Z(t)]=2\big\|[Z(t)]\big\|_1,\\Z(t)^H\eta(t)+\eta(t)^HZ(t)=2\|Z(t)\|_1\ge 2\|Z(t)\|_2.\end{gathered}$$

引理 4 [12] 对任意复数 $z\in\mathbb{C}$, 如下结论成立:

$$z+\bar{z}=2\mathrm{Re}(z)\le 2|z|_2\le 2|z|_1.$$

定义 2[14] 令 $V(x(t)):\mathbb{R}^n\to\mathbb{R}$ 是一个正定、正则且径向无界的函数, 并且 $x(t)$ 在 $[0,+\infty)$ 的任意闭区间上是绝对连续的, 那么 $x(t)$ 和 $V(x(t))$ 在 $t\in[0,+\infty)$ 上几乎处处可微, 并且

$$\frac{\mathrm{d}V(x(t))}{\mathrm{d}t}=\xi^T\dot{x}(t),\ \ \forall\xi\in\partial V(x(t)),$$

其中: $\partial V(x(t))$ 表示 $V(x(t))$ 的广义梯度.

引理 5 假设 $V(x(t)):\mathbb{R}^n\to\mathbb{R}^+$ 是一个正定、正则且径向无界的函数, 并且 $x(t)$ 在 $[0,+\infty)$ 的任意闭区间上是绝对连续的, 如果

$$\dot{V}(x(t))\le aV(x(t))-bV^{\beta}(x(t))-cV^{\theta}(x(t)),\ x(t)\in\mathbb{R}^n\setminus\{0\},$$

其中: $a\in\mathbb{R}$, $0\le\beta<1$, $\theta>1$, 并且正常数 b,c 满足 $\min\{b,c\}>a$, 则对任意 $t\ge T$, $V(x(t))=0$, $x(t)=0$, 这里 T 满足 $T\le\frac{1}{a(1-\beta)}\ln(\frac{b}{b-a})+\frac{1}{(c-a)(\theta-1)}$.

证明 令 $v(t)=V(x(t))$, 那么

$$\dot{v}(t)\le av(t)-bv^{\beta}(t)-cv^{\theta}(t). \tag{7}$$

由于 $0\le\beta<1,\theta>1$, $\min\{b,c\}>a$, 则 $\dot{v}(t)\le av(t)-bv^{\beta}(t)-cv^{\theta}(t)=-\Upsilon(v)\le 0$.

另一方面, 由 (7) 可得

$$T=\int_0^{v_0}\frac{1}{\Upsilon(v)}\,\mathrm{d}v=\frac{1}{1-\beta}\int_0^{v_0^{1-\beta}}\frac{\mathrm{d}v^{1-\beta}}{-av^{1-\beta}+b+cv^{\theta-\beta}},$$

这里 $v_0=v(0)$. 记 $\Psi=v^{1-\beta}$, $\Psi_0=v_0^{1-\beta}$. 若 $0<\Psi_0\le 1$, 那么

$$T\le\frac{1}{1-\beta}\int_0^{1}\frac{\mathrm{d}\Psi}{-a\Psi+b}=\frac{1}{a(1-\beta)}\ln(\frac{b}{b-a}).$$

如果 $\Psi_0>1$, 那么

$$\begin{aligned}T&=\frac{1}{1-\beta}\Big(\int_0^{1}\frac{\mathrm{d}\Psi}{-a\Psi+b+c\Psi^{\frac{\theta-\beta}{1-\beta}}}+\int_1^{\Psi_0}\frac{\mathrm{d}\Psi}{-a\Psi+b+c\Psi^{\frac{\theta-\beta}{1-\beta}}}\Big)\\&\le\frac{1}{1-\beta}\Big(\int_0^{1}\frac{\mathrm{d}\Psi}{-a\Psi+b}+\int_1^{\Psi_0}\frac{\mathrm{d}\Psi}{(-a+c)\Psi^{\frac{\theta-\beta}{1-\beta}}}\Big)\\&\le\frac{1}{a(1-\beta)}\ln(\frac{b}{b-a})+\frac{1}{c-a}\frac{1}{\theta-1}.\end{aligned}$$

综上, 不难得到

$$\int_0^{v_0}\frac{1}{\Upsilon(v)}\,\mathrm{d}v=T\le\frac{1}{a(1-\beta)}\ln(\frac{b}{b-a})+\frac{1}{c-a}\frac{1}{\theta-1}<+\infty.$$

由 Forti 等建立的有限时间稳定性定理[14] 可知, 对任意的 $t\ge T$, 有 $v(t)=0,x(t)=0$, 并且停息时间 T 不依赖于系统初始值.

2 主要结论

在本节中, 将通过设计一个不连续控制策略来讨论系统 (1) 和系统 (4) 的固定时间同步问题.

为了方便, 对任意的 $k=1,2,\cdots,n$, $i=1,2,\cdots,N$, 引入下列记号.

$$\tilde{a}^i_{kl}=\max\{|\acute{a}^i_{kl}|_1,|\grave{a}^i_{kl}|_1\},\bar{a}^i_{kl}=|\grave{a}^i_{kl}-\acute{a}^i_{kl}|_1,\bar{b}^i_{kl}=\max\{|\acute{b}^i_{kl}|_1,|\grave{b}^i_{kl}|_1\},$$

$$\tilde{a}_{ik}=\sum_{l=1}^{n}\tilde{a}^i_{lk}L_k,\ \ \bar{a}_{ik}=\sum_{l=1}^{n}\bar{a}^i_{kl}H_l,\ \bar{b}_{ik}=2\sum_{l=1}^{n}\bar{b}^i_{kl}H_l,$$

$$\tilde{d}_{ik}=\max\{|\mathrm{Im}(\acute{d}_{ik})|-\mathrm{Re}(\acute{d}_{ik}),|\mathrm{Im}(\grave{d}_{ik})|-\mathrm{Re}(\grave{d}_{ik})\},\quad \bar{d}_{ik}=|\grave{d}_{ik}-\acute{d}_{ik}|_1Q_{ik}.$$

$$M=\mathrm{diag}\{m_1,m_2,\cdots,m_N\},\ \ \tilde{D}_k=\mathrm{diag}\{\tilde{d}_{1k},\tilde{d}_{2k},\cdots,\tilde{d}_{Nk}\},$$

$$\bar{D}_k=\mathrm{diag}\{\bar{d}_{1k},\bar{d}_{2k},\cdots,\bar{d}_{Nk}\},\ \tilde{A}_k=\mathrm{diag}\{\tilde{a}_{1k},\tilde{a}_{2k},\cdots,\tilde{a}_{Nk}\},$$

$$\bar{A}_k=\mathrm{diag}\{\bar{a}_{1k},\bar{a}_{2k},\cdots,\bar{a}_{Nk}\},\ \bar{B}_k=\mathrm{diag}\{\bar{b}_{1k},\bar{b}_{2k},\cdots,\bar{b}_{Nk}\},$$

$$\bar{e}_k(t)=\big(|e_{1k}(t)|_1,|e_{2k}(t)|_1,\cdots,|e_{Nk}(t)|_1\big)^T,\ \bar{e}^*_k(t)=\big(\big|[e_{1k}(t)]\big|_1,\big|[e_{2k}(t)]\big|_1,\cdots,\big|[e_{Nk}(t)]\big|_1\big)^T.$$

定理 1 基于假设1, 从系统 (4) 中的控制器设计为

$$U_i(t)=-[e_i(t)]\big(m_i+\rho_1\|e_i(t)\|_1^{\beta}+\rho_2\|e_i(t)\|_1^{\theta}\big), \tag{8}$$

其中: $m_i\in\mathbb{R}$, $\rho_1,\rho_2>0$, $0\le\beta<1$, $\theta>1$, 若

$$\min\{\rho_1,N^{1-\theta}\rho_2\}>\vartheta_1,\ \ \bar{D}_k+\bar{A}_k+\bar{B}_k-M\le 0,\ \ k=1,2,\cdots,n, \tag{9}$$

这里 $\vartheta_1=\max_{1\le k\le n}\{\lambda_{\max}\big(\tilde{D}_k+\tilde{A}_k+c\nu_k\Omega\big)\}$, 则耦合 MCNNs (1) 和 (4) 是固定时间同步的. 此外, 同步停息时间 T_0 满足

$$T_0\le T_0^*=\frac{1}{(1-\beta)\vartheta_1}\ln\frac{\rho_1}{\rho_1-\vartheta_1}+\frac{1}{\theta-1}\frac{1}{N^{1-\theta}\rho_2-\vartheta_1}.$$

证明 由可测选择理论[11], 存在向量 $\eta_i(t)=(\eta_{i1}(t),\cdots,\eta_{in}(t))^T$, $\eta_{ik}(t)\in\overline{co}\big([e_{ik}(t)]\big)$, 使得对几乎所有的 $t\ge 0$, 任意$i=1,2\cdots,N$,

$$\begin{aligned}\dot{e}_i(t)=&-\check{D}_iy_i(t)+\hat{D}_i(t)x_i(t)+\check{A}_if(y_i(t))-\hat{A}_if(x_i(t))+\check{B}_if(y_i(t-\tau(t)))\\&-\hat{B}_if(x_i(t-\tau(t)))+c\sum_{i=1}^{N}\omega_{ij}\tilde{g}(e_j(t))-\eta_i(t)\big(m_i+\rho_1\|e_i(t)\|_1^{\beta}+\rho_2\|e_i(t)\|_1^{\theta}\big).\end{aligned}\tag{10}$$

考虑如下 Lyapunov 函数

$$V_1(e(t))=\sum_{i=1}^{N}\|e_i(t)\|_1=\frac{1}{2}\sum_{i=1}^{N}\big([e_i(t)]^He_i(t)+e_i(t)^H[e_i(t)]\big),$$

其中: $e(t)=(\|e_1(t)\|_1,\|e_2(t)\|_1,\cdots,\|e_N(t)\|_1)^T$. 当$e(t)\in\mathbb{R}^N\setminus\{0\}$时，

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}V_1(e(t))=&\frac{1}{2}\sum_{i=1}^{N}\big([e_i(t)]^H\dot{e}_i(t)+\dot{e}_i^H(t)[e_i(t)]\big)\\
=&\frac{1}{2}\sum_{i=1}^{N}\Big([e_i(t)]^H\big(-\check{D}_iy_i(t)+\hat{D}_ix_i(t)\big)+\big(-\check{D}_iy_i(t)+\hat{D}_ix_i(t)\big)^H[e_i(t)]\Big)\\
&+\frac{1}{2}\sum_{i=1}^{N}\Big([e_i(t)]^H\big(\check{A}_if(y_i(t))-\hat{A}_if(x_i(t))\big)+\big(\check{A}_if(y_i(t))-\hat{A}_if(x_i(t))\big)^H[e_i(t)]\Big)\\
&+\frac{1}{2}\sum_{i=1}^{N}[e_i(t)]^H\big(\check{B}_if(y_i(t-\tau(t)))-\hat{B}_if(x_i(t-\tau(t)))\big)\\
&+\frac{1}{2}\sum_{i=1}^{N}\big(\check{B}_if(y_i(t-\tau(t)))-\hat{B}_if(x_i(t-\tau(t)))\big)^H[e_i(t)]\\
&+\frac{c}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\big([e_i(t)]^H\omega_{ij}\tilde{g}(e_j(t))+\tilde{g}^H(e_j(t))\omega_{ij}[e_i(t)]\big)\\
&-\frac{1}{2}\sum_{i=1}^{N}\big([e_i(t)]^H\eta_i(t)+\eta_i^H(t)[e_i(t)]\big)\big(m_i+\rho_1\|e_i(t)\|_1^{\beta}+\rho_2\|e_i(t)\|_1^{\theta}\big).\end{aligned}\tag{11}$$

由忆阻连接权重的定义及分类讨论可得

$$\begin{aligned}&\frac{1}{2}\sum_{i=1}^{N}\Big([e_i(t)]^H\big(-\check{D}_iy_i(t)+\hat{D}_ix_i(t)\big)+\big(-\check{D}_iy_i(t)+\hat{D}_ix_i(t)\big)^H[e_i(t)]\Big)\\
&\le\sum_{i=1}^{N}\sum_{k=1}^{n}\Big(\tilde{d}_{ik}\big|e_{ik}(t)\big|_1+\bar{d}_{ik}\big|[e_{ik}(t)]\big|_1\Big).\end{aligned}\tag{12}$$

同理, 基于假设 1 不难得到,

$$\begin{aligned}&\frac{1}{2}\sum_{i=1}^{N}\big([e_i(t)]^H[\check{A}_if(y_i(t))-\hat{A}_if(x_i(t))]+[\check{A}_if(y_i(t))-\hat{A}_if(x_i(t))]^H[e_i(t)]\big)\\
&\le\sum_{i=1}^{N}\sum_{k=1}^{n}\sum_{l=1}^{n}(\tilde{a}_{kl}^iL_l\big|e_{il}(t)\big|_1+\bar{a}_{kl}^iH_l\big|[e_{ik}(t)]\big|_1)\\
&\le\sum_{i=1}^{N}\sum_{k=1}^{n}\Big(\tilde{a}_{ik}|e_{ik}(t)|_1+\bar{a}_{ik}|[e_{ik}(t)]|_1\Big),\end{aligned}\tag{13}$$

$$\begin{aligned}
&\frac{1}{2}\sum_{i=1}^{N}[e_i(t)]^H\big(\check{B}_i f(y_i(t-\tau(t)))-\hat{B}_i f(x_i(t-\tau(t)))\big)\\
&+\frac{1}{2}\sum_{i=1}^{N}\big(\check{B}_i f(y_i(t-\tau(t)))-\hat{B}_i f(x_i(t-\tau(t)))\big)^H[e_i(t)]\\
&\le 2\sum_{i=1}^{N}\sum_{k=1}^{n}\sum_{l=1}^{n}\bar{b}^i_{kl}H_l\big|[e_{ik}(t)]\big|_1\\
&\le \sum_{i=1}^{N}\sum_{k=1}^{n}\bar{b}_{ik}|[e_{ik}(t)]|_1.
\end{aligned}\tag{14}$$

由假设 1 可知

$$\begin{aligned}
&\frac{c}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\big([e_i(t)]^H\omega_{ij}\tilde{g}(e_j(t))+\tilde{g}^H(e_j(t))\omega_{ij}[e_i(t)]\big)=c\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{n}\omega_{ij}\mathrm{Re}(\overline{[e_{ik}(t)]}g_k(e_{jk}(t)))\\
&\le c\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{n}\omega_{ij}\nu_k\big|e_{jk}(t)\big|_1.
\end{aligned}\tag{15}$$

基于引理 3, 我们可以得到

$$\begin{aligned}
&-\frac{1}{2}\sum_{i=1}^{N}([e_i(t)]^H\eta_i(t)+\eta_i^H(t)[e_i(t)])\big(m_i+\rho_1\|e_i(t)\|_1^{\beta}+\rho_2\|e_i(t)\|_1^{\theta}\big)\\
&\le -\sum_{i=1}^{N}\sum_{k=1}^{n}\big|[e_{ik}(t)]\big|_1 m_i-\sum_{i=1}^{N}\big\|[e_i(t)]\big\|_1\big(\rho_1\|e_i(t)\|_1^{\beta}+\rho_2\|e_i(t)\|_1^{\theta}\big)\\
&\le -\sum_{i=1}^{N}\sum_{k=1}^{n}\big|[e_{ik}(t)]\big|_1 m_i-\sum_{i=1}^{N}\big(\rho_1\|e_i(t)\|_1^{\beta}+\rho_2\|e_i(t)\|_1^{\theta}\big).
\end{aligned}\tag{16}$$

将式(12)~(16) 代入 (11), 由引理 1 及条件 (9) 可得

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}V_1(e(t))\le&\sum_{k=1}^{n}I_N^T\big(\tilde{D}_k+\tilde{A}_k+c\nu_k\Omega\big)\bar{e}_k(t)\\
&+\sum_{k=1}^{n}I_N^T\big(\bar{D}_k+\bar{A}_k+\bar{B}_k-M\big)\bar{e}_k^*(t)-\sum_{i=1}^{N}(\rho_1\|e_i(t)\|_1^{\beta}+\rho_2\|e_i(t)\|_1^{\theta})\\
&\le\vartheta_1 V(e(t))-\rho_1V^{\beta}(e(t))-\rho_2N^{1-\theta}V^{\theta}(e(t)).
\end{aligned}\tag{17}$$

由引理 5 可知, 系统 (1) 和 (4) 在控制器 (8) 下是固定时间同步的, 且停息时间 T_0 满足

$$T_0\le T_0^*=\frac{1}{(1-\beta)\vartheta_1}\ln\frac{\rho_1}{\rho_1-\vartheta_1}+\frac{1}{\theta-1}\frac{1}{N^{1-\theta}\rho_2-\vartheta_1}.$$

证明完毕.

当系统 (1) 和 (4) 中的耦合函数退化为线性情况时, 即对任意的$v\in\mathbb{C}^n$, $g(v)=\Gamma v$, $\Gamma=\mathrm{diag}\{\gamma_1,\gamma_2,\cdots,\gamma_n\}$, $\gamma_k\in\mathrm{R}$, 我们不难得到以下结论.

推论 1 在假设 1 及控制器 (8) 下, 若

$$\min\{\rho_1,\ N^{1-\alpha}\rho_2\}>\vartheta_1^*,\ \bar{D}_k+\bar{A}_k+\bar{B}_k-M\le 0,\ \ k=1,2,\cdots,n,$$

其中: $\vartheta_1^*=\max_{1\le k\le n}\{\lambda_{\max}(\tilde{D}_k+\tilde{A}_k+c\gamma_k\Omega)\}$, 则耦合 MCNNs (1) 和 (4) 是固定时间同步的. 此外, 同步的停息时间 T_1 满足

$$T_1\le T_1^*=\frac{1}{(1-\beta)\vartheta_1^*}\ln\frac{\rho_1}{\rho_1-\vartheta_1^*}+\frac{1}{\theta-1}\frac{1}{N^{1-\theta}\rho_2-\vartheta_1^*}.$$

注 1 在定理 1 中, 如果 $\beta=0$, T_0 的估计值将退化为如下形式

$$T_0 \le T_2^* = \frac{1}{\vartheta_1}\ln\frac{\rho_1}{\rho_1-\vartheta_1} + \frac{1}{\theta-1}\frac{1}{N^{1-\theta}\rho_2-\vartheta_1}.$$

注 2 在以往关于 MCNNs 动力学分析的理论研究中[4,5], 其主要结论是通过将复值系统事先分离成两个实值子系统, 再进行理论分析而得到的. 不同于这种分离方法, 本文复值符号函数的引入使得控制器设计、Lyapunov 函数的选取以及理论分析均在复数域内进行.

注 3 在大多数关于固定时间同步的研究中[6,7], 关于同步误差的线性项在控制器设计中是必不可少的. 与此不同, 本文给出的控制策略移除了该线性部分, 简化了传统的控制设计. 另外, 引理 5将现有的固定时间稳定性推广到不连续系统中, 并提高了已有工作[8]中给出的停息时间估计精度.

3 数值模拟

本部分将通过一个数值实例说明所得结论的有效性.

在系统 (1) 和 (4) 中, 选取网络节点数为 $N=12$, $n=2$, $\Pi_{ik}=\{x_{ik}(t)\in\mathbb{C};\ |\mathrm{Re}(x_{ik}(t))|+|\mathrm{Im}(x_{ik}(t))|\le 1.5,\ k=1,2\}$, $f_k(x_{ik}(t))=\tanh(\mathrm{Re}(x_{ik}(t)))+\mathbf{i}\sin(\mathrm{Im}(x_{ik}(t)))$, $k=1,2$, 忆阻连接权重定义为: 当 $x_{ik}(t),y_{ik}(t)\in\Pi_{ik}$, $d_{i1}=1.07+0.22\mathbf{i}, d_{i2}=0.49-0.60\mathbf{i}$, $a_{11}^i=2.20-1.03\mathbf{i}, a_{12}^i=-0.14+0.60\mathbf{i}, a_{21}^i=-5.10+3.49\mathbf{i}, a_{22}^i=3.00-2.02\mathbf{i}$ 并且 $b_{11}^i=-1.59+0.10\mathbf{i}, b_{12}^i=-1.19-0.13\mathbf{i}, b_{21}^i=-0.10-0.42\mathbf{i}, b_{22}^i=-2.10-1.90\mathbf{i}$; 当 $x_{ik}(t),y_{ik}(t)\notin\Pi_{ik}$, $d_{i1}=0.98+0.12\mathbf{i}, d_{i2}=0.51-0.62\mathbf{i}$, $a_{11}^i=2.10-0.99\mathbf{i}, a_{12}^i=-0.11+0.56\mathbf{i}, a_{21}^i=-5.12+3.51\mathbf{i}, a_{22}^i=3.10-2.08\mathbf{i}$, 并且 $b_{11}^i=-1.65+0.15\mathbf{i}, b_{12}^i=-1.11-0.12\mathbf{i}, b_{21}^i=-0.19-0.40\mathbf{i}, b_{22}^i=-1.99-1.89\mathbf{i}$.

基于以上参数, 系统 (1) 中孤立节点的实部和虚部动力学行为模拟如图1和图2所示, 这里 $x_1(0)=-2.18-1.09\mathbf{i}$, $x_2(0)=-1.97+1.22\mathbf{i}$.

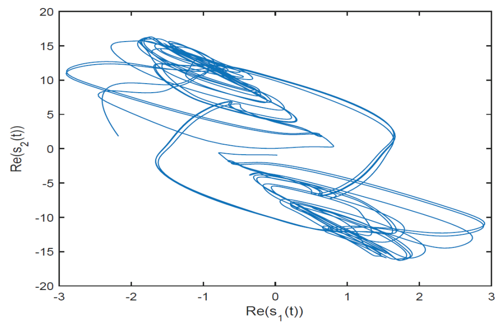

图 1 孤立节点的实部相

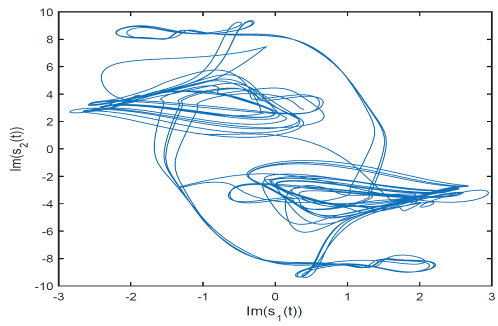

图 2 孤立节点的虚部相图

对任意的 $i=1,2,\cdots,12$, $k=1,2$, 选取 $g_k(x_{ik}(t))=\tanh(\mathrm{Re}(x_{ik}(t)))+\mathbf{i}\tanh(\mathrm{Im}(x_{ik}(t)))$. 通过简要计算可得, $L_k=\nu_k=1$, $H_k=2$. 在控制器 (8) 中, 选取下列控制参数: $m_i=14.70$, $\rho_1=11.20$, $\rho_2=34.26$, $\theta=1.42$. 则由定理 1 可知, 耦合 MCNNs (1) 和 (4) 是固定时间同步的, 如图3和图4所示. 此外, 对应的停息时间估计分别为 $T_0^*=22.00$, $T_2^*=21.82$.

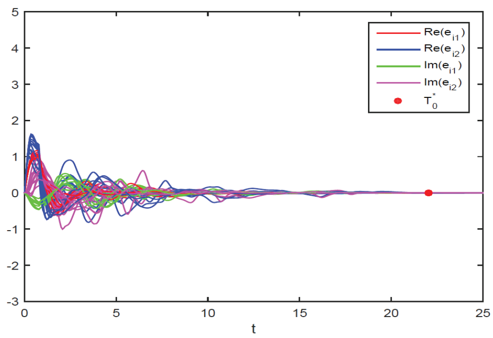

图 3 $\beta=0.5$ 时的同步误差演化

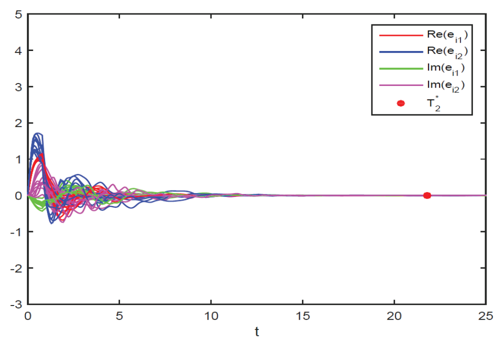

图 4 $\beta=0$ 时的同步误差演化

4 结 论

本文主要讨论了一类具有时滞的耦合忆阻复值神经网络的固定时间同步. 基于引入的复值符号函数和新的固定时间稳定性理论, 设计了新的不连续复值控制策略, 并通过构造新的 Lyapunov 函数来分析网络的固定时间同步. 此外, 所提出的控制方法通过移除线性部分简化了传统控制设计. 未来工作将研究脉冲系统或时空网络的固定时间控制问题.

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