Exponential Synchronization of Competitive Neural Networks with Time Delay under Intermittent Control*

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Abstract: In this paper, the problem about the exponential synchronization of competitive neural networks with time delay under intermittent control is discussed. By using Lyapunov function method and differential inequality method, some sufficient conditions are given which are based on global exponential synchronization of competitive neural networks with large time delay and small time delay, and the corresponding criterion is established. Finally, a numerical example is shown to verify the correctness and validity of the theoretical result.

Key words: competitive neural networks; synchronization; time-varying delay; intermittent control

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0 Introduction

Competitive neural networks model is an important kind of unsupervised synaptic modification neural network, which contains two state variables: short-term memory variable (STM) to describe fast neural activities, and long-term memory variable (LTM) to describe slow neural activities. Since Cohen and Grossberg [1] first proposed the competitive neural networks in 1983, there have been many works about the competitive neural networks, such as competing neural networks with different time scales [2−5], time-varying distributed competitive neural networks with time delay [6,7], unsupervised competitive neural networks [8], competitive neural networks with different time scales and random disturbances [9], delay competitive neural networks with discontinuous activation function [10], competitive neural networks with different time scales and time-varying delays [11].

In addition, synchronization is a typical collective behavior, which is widely used in biological evolution, chemical reaction, safety communication and so on. Since the pioneering works of Pecora and Carroll [12], and the synchronization of competitive neural networks has attracted more and more attentions. In [4], Lou and Cui studied the exponential synchronization of a class of competitive neural networks, and designed some exponential synchronization criteria by using Lyapunov function, linear matrix inequality method and Newton-Leibniz formula. In [9], an adaptive feedback controller was designed by Gu to realize the complete synchronization of the coupled time-delay competitive neural networks with different time scales and random disturbances. By using the LaSalle invariance principle of stochastic differential delay equations, the globally certain asymptotic stability of error dynamic systems was studied. In [11], Gan studied the synchronization of a class of competitive neural networks with different time scales and time-varying delays. He proposed a new delay partition method and derived a delay correlation condition to ensure the synchronization of response system and drive system. By solving LMI, the design of gain matrix of linear feedback controller can be realized. In [13], Zhang et al studied the synchronization problem of dynamic networks under hybrid impulse and switch control. The results show that collective synchronous motion appeared in many large-scale dynamical networks. Based on the concept of impulse control and the stability theory of the impulse system, a new criterion of impulse synchronization for the model was established.

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In recent years, discontinuous feedback control strategies such as intermittent control have been widely used in traffic systems, communication and other fileds [14–16]. Therefore, it is necessary to study the intermittent control methods. For example, in [17], periodic intermittent nonlinear feedback control method is used to synchronize a class of nonlinear chaotic systems with time delay. Based on Lyapunov functional theory and Halaner inequality, some synchronization criteria were derived and sufficient conditions were obtained to guarantee delay-free synchronization of nonlinear systems. In [18], the problem of exponential stability for a class of nonlinear systems was studied by means of periodic intermittent control. In this paper, Li et al. gave a set of exponential stability criteria for linear matrix inequalities, and a simple sufficient condition determined by three scalar inequalities. On the premise that the control period and control width were fixed and known, a suboptimal intermittent controller was designed for the general cost function and the theoretical results were verified by numerical simulation. In [19], the exponential stability of delayed chaotic neural network was studied by using periodic intermittent control method. Using Lyapunov function and Halaner inequality, the authors established the exponential stability criterion and its simplified form of the controlled neural networks. The feasible region of control parameters was estimated strictly. The theoretical results and numerical simulation show that the continuous time DCNN could be stabilized by using the non-zero duration intermittent feedback control.

Based on the above discussion, we consider the competitive neural networks model with time-varying delay. By using Lyapunov function method and differential inequality analysis method, the global exponential synchronization conditions of competitive neural networks with time delay are discussed, and the corresponding criteria are established. At the same time, we studied the exponential synchronization problems of competitive neural networks with both large and small time delays and intermittent control, and some sufficient conditions are established. Finally, a numerical example is shown to verify the correctness and validity of the theoretical result.

1 Model building and preliminaries

Consider the following competitive neural networks with time delay

$$\begin{cases} STM: \varepsilon\dot{x}_i(t) = -a_i x_i(t) + \sum\limits_{k=1}^{N} d_{ik} f_k(x_k(t)) + \sum\limits_{k=1}^{N} d_{ik}^{\tau} f_k(x_k(t-\tau)) + b_i \sum\limits_{j=1}^{P} m_{ij}(t)\sigma_j, \\ LTM: \dot{m}_{ij}(t) = -c_i m_{ij}(t) + \sigma_j f_i(x_i(t)), \end{cases} \tag{1}$$

where N is the number of junction neurons in STM state, P is the number of neurons stimulated by external stimulation, x_i indicates the current activation level of neurons, $f_i(\cdot)$ represents the neuron output function, $m_{ij}(t)$ is recorded as a valid join, σ_j is recorded as external stimulus weight, $a_i > 0$ is called the neuron activation constant , $c_i > 0$ is any constant, d_{ik} represents the connection weight between the i-th and k-th neurons, b_i represents the degree of external stimulation, $\varepsilon > 0$ represents the time scale in the STM state, τ is a delay constant: d_{ik}^{τ} represents the connection weight of the delay feedback between the i-th and k-th neurons.

Let $s_i(t) = \sum\limits_{j=1}^{P} m_{ij}(t)\sigma_j = m_i(t)\sigma$, where $\sigma = (\sigma_1, \sigma_2, \cdots, \sigma_P)^T$, $m_i(t) = (m_{i1}(t), m_{i2}(t), \cdots, m_{iP}(t))^T$, then the system (1) can be represented as follows

$$\begin{cases} STM: \varepsilon\dot{x}_i(t) = -a_i x_i(t) + \sum\limits_{k=1}^{N} d_{ik} f_k(x_k(t)) + \sum\limits_{k=1}^{N} d_{ik}^{\tau} f_k(x_k(t-\tau)) + b_i s_i(t), \\ LTM: \dot{s}_i(t) = -c_i s_i(t) + |\sigma|^2 f_i(x_i(t)), \end{cases} \tag{2}$$

where $|\sigma|^2 = \sigma_1^2 + \sigma_1^2 + \cdots + \sigma_P^2$ is a constant, without loss of generality, let $|\sigma|^2 = 1$, and when $t \in \mathbb{R}^+ = [0, +\infty)$, let $x(t) = (x_1(t), x_2(t), \cdots, x_N(t))^T$, $S(t) = (s_1(t), s_2(t), \cdots, s_N(t))^T$, $A = \mathrm{diag}(a_1, a_2, \cdots, a_N)$, $D = (d_{ik})_{N\times N}$, $D_\tau = (d_{ik}^{\tau})_{N\times N}$, $B = \mathrm{diag}(b_1, b_2, \cdots, b_N)$, $C = \mathrm{diag}(c_1, c_2, \cdots, c_N)$, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \cdots, f_N(x_N(t)))^T$, $f(x(t-\tau)) = (f_1(x_1(t-\tau)), f_2(x_2(t-\tau)), \cdots, f_N(x_N(t-\tau)))^T$. So the matrix form of system (2) is

$$\begin{cases} STM: \varepsilon\dot{x}(t) = -Ax(t) + Df(x(t)) + D_\tau f(x(t-\tau)) + BS(t), \\ LTM: \dot{S}(t) = -CS(t) + f(x(t)). \end{cases} \tag{3}$$

Therefore, the system is described as follows

$$\begin{cases} STM: \varepsilon\dot{y}(t) = -Ay(t) + Df(y(t)) + D_\tau f(y(t-\tau)) + BR(t) + u_1(t), \\ LTM: \dot{R}(t) = -CR(t) + f(y(t)) + u_2(t), \end{cases} \tag{4}$$

where $y(t) = (y_1(t), y_2(t), \cdots, y_N(t))^T$, $R(t) = (r_1(t), r_2(t), \cdots, r_N(t))^T$, $u_i(t)(i=1,2)$ is an intermittent controller

$$u_1(t) = \begin{cases} -Q_1 e(t), & nT \le t < nT + \delta, \\ 0, & nT + \delta \le t < (n+1)T, \end{cases} \tag{5}$$

$$u_2(t) = \begin{cases} -Q_2 z(t), & nT \le t < nT + \delta, \\ 0, & nT + \delta \le t < (n+1)T, \end{cases} \tag{6}$$

$Q_i \in R^{N\times N}$ $(i=1,2)$ is the gain matrix of feedback control. Let $e(t) = y(t) - x(t), z(t) = R(t) - S(t)$, then the error dynamic system can be written as

$$\begin{cases} STM: \varepsilon\dot{e}(t) = -Ae(t) + Dg(e(t)) + D_\tau g(e(t-\tau)) + Bz(t) + u_1(t), \\ LTM: \dot{z}(t) = -Cz(t) + g(e(t)) + u_2(t), \end{cases} \tag{7}$$

where $g(e(t)) = f(y(t)) - f(x(t))$, $g(e(t-\tau)) = f(y(t-\tau)) - f(x(t-\tau))$.

Assuming 1 There is a symmetric positive definite matrix $L = (l_{ij})_{m\times n}$, for any $x, y \in R^n$, we have $\|f(x) - f(y)\| \le L\|x - y\|$.

Assuming 2 Time-lag $\tau < T$, where τ is a constant, T is a cycle.

Lemma 1[18] For any real matrix with proper dimension $\Sigma_1, \Sigma_2, \Sigma_3$, if given $0 < \Sigma_3 = \Sigma_3^T$ and any real number $\varepsilon > 0$, then $\Sigma_1^T\Sigma_2 + \Sigma_2^T\Sigma_1 \le \varepsilon\Sigma_1^T\Sigma_3\Sigma_1 + \varepsilon^{-1}\Sigma_2^T\Sigma_3^{-1}\Sigma_2$ holds.

Lemma 2[13] If $P \in R^{m\times n}$ is a positive definite matrix, Q is a symmetric matrix, then we have $\lambda_{\min}(P^{-1}Q)X^TPX \le X^TQX \le \lambda_{\max}(P^{-1}Q)X^TPX, X \in R^n$.

Lemma 3[17] Let φ is a nonnegative function in interval $[t_0 - \tau, +\infty)$, and it is continuous in the subinterval $[t_0, +\infty)$. When $t \ge t_0$, $\varphi' \le \alpha\varphi(t) + \beta\varphi(t-\tau)$ holds. If $\alpha > 0, \beta > 0$, then, when $t \ge t_0$, we have $\varphi(t) \le \varphi_{t_0} e^{\eta(t-t_0+\tau)}$, where $\varphi_{t_0} = \sup_{t_0-\tau\le\theta\le t_0}\varphi(\theta)$, and $\eta > 0$ is the unique solution of the equation $\alpha + \beta e^{-\eta\tau} = \eta$.

Lemma 4(Halanay inequality)[19] Let $\varphi : [t_0 - \tau) \to [0, +\infty)$ is a continuous function, when $t \ge t_0$, such that $\varphi' \le -\alpha\varphi(t) + \beta\varphi_t$. If $\alpha > 0, \beta > 0$, then, when $t \ge t_0$, we have $\varphi(t) \le \varphi_{t_0} e^{-\gamma(t-t_0)}$, where $\varphi_t = \sup_{t-\tau\le\theta\le t}\varphi(\theta)$, and $\gamma > 0$ is the unique solution of the equation $\alpha - \gamma - \beta e^{\gamma\tau} = 0$.

Lemma 5 Suppose the function $y(t)$ is continuous and nonnegative in $t \in [-\tau, +\infty)$. If there are constants γ_1, γ_2, γ_3, γ_4 the following conditions are met

$$\begin{cases} \dot{y}(t) \le -\gamma_1 y(t) + \gamma_2 y(t-\tau), & t \in [nT, nT+\delta), \\ \dot{y}(t) \le \gamma_3 y(t) + \gamma_4 y(t-\tau), & t \in [nT+\delta, (n+1)T), \end{cases} \tag{8}$$

and $\gamma_1 > \bar{\gamma} \ge 0$, $\bar{\eta} = \gamma_1 + \gamma_3 > 0$, $\rho = \lambda^* - \bar{\eta}(1 - \frac{\delta}{T}) > 0$, then $y(t) \le \sup\limits_{-\tau\le s\le 0} y(s)\exp\{-\rho t\}$, where $0 < \delta < T$, $\bar{\gamma} = \max\{\gamma_2, \gamma_4\}$, λ^* is the solution of the equation $\lambda^* - \gamma_1 + \bar{\gamma}\exp\{\lambda^*\tau\} = 0$.

Proof Suppose $\mathcal{W}(t) = y(t)\exp\{\lambda^* t\}, t \ge 0$; $\mathcal{M}_0 = \sup\limits_{-\tau\le s\le 0} y(s)$, where λ^* is the solution of the equation $\lambda^* - \gamma_1 + \bar{\gamma}\exp\{\lambda^*\tau\} = 0$. By lemma 4, we obtain $\mathcal{W}(t) \le \mathcal{M}_0$. So, for any $t \in [0, \delta)$, we have $\mathcal{W}(t) < h\mathcal{M}_0$, where constant $h > 1$. Then, we need to prove for any $t \in [\delta, T)$, the formula $\mathcal{W}(t) < h\mathcal{M}_0\exp\{\bar{\eta}(t-\delta)\}$ holds, i.e.,

$$\mathcal{H}(t) = \mathcal{W}(t) - h\mathcal{M}_0\exp\{\bar{\eta}(t-\delta)\} < 0. \tag{9}$$

Otherwise, there exists $p_0 \in (\delta, T)$, such that

$$\mathcal{H}(p_0) = 0,\ \ \dot{\mathcal{H}}(p_0) \ge 0,\ \ \mathcal{H}(t) < 0,\ \ t \in (\delta, p_0). \tag{10}$$

For $\tau > 0$, if $\delta < p_0 - \tau(p_0) < p_0$, then $\mathcal{W}(p_0 - \tau(p_0)) < h\mathcal{M}_0 \exp\{\bar{\eta}(p_0 - \delta)\}$. In addition, if $-\tau \le p_0 - \tau(p_0) \le \delta$, we have $\mathcal{W}(p_0 - \tau(p_0)) < h\mathcal{M}_0 < h\mathcal{M}_0 \exp\{\bar{\eta}(p_0 - \delta)\}$. Therefore, for any τ, we have $\mathcal{W}(p_0 - \tau(p_0)) < h\mathcal{M}_0 \exp\{\bar{\eta}(p_0 - \delta)\}$. Hence,

$$\begin{aligned}\dot{\mathcal{H}}(p_0) &= \lambda^* \mathcal{W}(p_0) + \exp\{\lambda^* p_0\}\dot{y}(p_0) - \bar{\eta} h\mathcal{M}_0 \exp\{\bar{\eta}(p_0 - \delta)\} \\ &= (\lambda^* + \gamma_3)\mathcal{W}(p_0) + \gamma_4 \exp\{\lambda^* \tau\}\mathcal{W}(p_0 - \tau) - \bar{\eta} h\mathcal{M}_0 \exp\{\bar{\eta}(p_0 - \delta)\} \\ &< (\lambda^* + \gamma_3 + \gamma_4 \exp\{\lambda^* \tau\} - \bar{\eta}) h\mathcal{M}_0 \exp\{\bar{\eta}(p_0 - \delta)\} \\ &\le (\lambda^* - \gamma_1 + \bar{\gamma} \exp\{\lambda^* \tau\}) h\mathcal{M}_0 \exp\{\bar{\eta}(p_0 - \delta)\} = 0 \end{aligned} \tag{11}$$

This contradicts with the inequality (10). So, the inequality (9) holds, i.e., for any $t \in [\delta, T)$,

$$\mathcal{W}(t) < h\mathcal{M}_0 \exp\{\bar{\eta}(t - \delta)\} < h\mathcal{M}_0 \exp\{\bar{\eta}(T - \delta)\}. \tag{12}$$

Then, for any $t \in [-\tau, T)$, we have $\mathcal{W}(t) < h\mathcal{M}_0 \exp\{\bar{\eta}(T - \delta)\}$.

Next, we prove for any $t \in [T, T + \delta)$, we have

$$\mathcal{W}(t) < h\mathcal{M}_0 \exp\{\bar{\eta}(T - \delta)\}. \tag{13}$$

Otherwise, there exists $p_1 \in [T, T + \delta)$, such that

$$\tilde{\mathcal{H}}(p_1) = 0, \ \ \dot{\tilde{\mathcal{H}}}(p_1) \ge 0, \ \ \tilde{\mathcal{H}}(t) < 0, \ \ t \in (t_1, p_1), \tag{14}$$

where $\tilde{\mathcal{H}}(t) = \mathcal{W}(t) - h\mathcal{M}_0 \exp\{\bar{\eta}(T - \delta)\}$. So

$$\begin{aligned}\dot{\tilde{\mathcal{H}}}(p_1) &= \lambda^* \mathcal{W}(p_1) + \exp\{\lambda^* p_1\}\dot{y}(p_1) \\ &= (\lambda^* - \gamma_1)\mathcal{W}(p_1) + \gamma_2 \exp\{\lambda^* \tau(p_1)\}\mathcal{W}(p_1 - \tau) \\ &< (\lambda^* - \gamma_1 + \gamma_2 \exp\{\lambda^* \tau\}) h\mathcal{M}_0 \exp\{\bar{\eta}(T - \delta)\} \\ &\le (\lambda^* - \gamma_1 + \bar{\gamma} \exp\{\lambda^* \tau\}) h\mathcal{M}_0 \exp\{\bar{\eta}(T - \delta)\} = 0. \end{aligned} \tag{15}$$

This contradicts with the inequality (14). Hence, (13) holds.

For $t \in [T + \delta, 2T)$, by the similar method, we get

$$\mathcal{W}(t) < h\mathcal{M}_0 \exp\{\bar{\eta}[(T - \delta) + (t - (T + \delta))]\}.$$

Repeating the above process, for any $t_k \le t \le s_k$, we have

$$\mathcal{W}(t) < h\mathcal{M}_0 \exp\{\bar{\eta} k(T - \delta)\}. \tag{16}$$

In addition, for $kT + \delta < t < (k+1)T$, then

$$\mathcal{W}(t) < h\mathcal{M}_0 \exp\{\bar{\eta}[k(T - \delta) + (t - kT + \delta)]\}. \tag{17}$$

For any $t \ge 0$, there exists a positive integer k, such that $kT \le t \le (k+1)T$. So, for any t, we have the following estimations for $\mathcal{W}(t)$. When $kT \le t < kT + \delta$,

$$\begin{aligned}\mathcal{W}(t) &< h\mathcal{M}_0 \exp\{\bar{\eta} k(T - \delta)\} \\ &= h\mathcal{M}_0 \exp\left\{\bar{\eta} k \frac{T - \delta}{T} \cdot T\right\} \\ &= h\mathcal{M}_0 \exp\left\{\bar{\eta}\left(1 - \frac{\delta}{T}\right) kT\right\} \\ &\le h\mathcal{M}_0 \exp\left\{\bar{\eta}\left(1 - \frac{\delta}{T}\right) t\right\}. \end{aligned}$$

When $kT+\delta\le t<(k+1)T$, we have

$$\begin{aligned}\mathcal{W}(t)&<h\mathcal{M}_0\exp\{\bar{\eta}[k(T-\delta)+(t-kT-\delta)]\}\\&=h\mathcal{M}_0\exp\left\{\bar{\eta}\left[k\frac{T-\delta}{T}\cdot T+\frac{t-kT-\delta}{t-kT}\cdot(t-kT)\right]\right\}\\&\le h\mathcal{M}_0\exp\left\{\bar{\eta}\left[k\left(1-\frac{\delta}{T}\right)kT+\frac{\delta}{T}\cdot(t-kT)\right]\right\}\\&\le h\mathcal{M}_0\exp\left\{\bar{\eta}\left(1-\frac{\delta}{T}\right)[kT+(t-kT)]\right\}\\&=h\mathcal{M}_0\exp\left\{\bar{\eta}\left(1-\frac{\delta}{T}\right)t\right\}.\end{aligned}$$

Let $h\to 1$, by the definition of $\mathcal{W}(t)$, we have $y(t)\le\mathcal{M}_0\exp\{-(\lambda^*-\bar{\eta}(1-\frac{\delta}{T}))t\}=\mathcal{M}_0\exp\{-\rho t\},\quad t\ge 0.$

Remark 1 In lemma 5, we only demand $\tau<T$. There is no limit to the size relationship between δ and τ, i.e., lemma 5 holds for $\tau<\delta$ and $\tau\ge\delta$.

2 Synchronization criterion of competitive neural networks under intermittent control

In this section, the strict derivation of the synchronization criterion for system (3) and system (4) is given based on differential inequality and intermittent control strategy.

Theorem 1 Under assumings 1, 2, the system (3) and system (4) synchronize with intermittent control protocol, if there exists positive numbers α_1, α_2, ε_i, $\mu_i (i=1,\cdots,4)$ and positive definite matrices Q_1, Q_2 meeting the following conditions

(1)$-2A-2Q_1+\varepsilon_1^{-1}D^TD+\varepsilon_1L^2+\varepsilon_2^{-1}D_\tau^TD_\tau+\varepsilon\varepsilon_3L^2+\varepsilon_4^{-1}I+\varepsilon\alpha_1I\le 0$,

(2)$-2C-2Q_2+\varepsilon^{-1}\varepsilon_3^{-1}I+\varepsilon^{-1}\varepsilon_4B^TB+\alpha_1I\le 0$,

(3)$-2A+\mu_1^{-1}D^TD+\mu_1L^2+\mu_2^{-1}D_\tau^TD_\tau+\varepsilon\mu_3L^2+\mu_4^{-1}I-\varepsilon\alpha_2I\le 0$,

(4)$-2C+\varepsilon^{-1}\mu_3^{-1}I+\varepsilon^{-1}\mu_4B^TB-\alpha_2I\le 0$,

(5) (i) If $0<\tau<\delta<T,\gamma(\delta-\tau)-\eta(T-\delta+\tau)>0$,

(ii) If $0<\delta<\tau<T,\alpha_1>\bar{\gamma}\ge 0,\bar{\eta}=\alpha_1+\alpha_2>0,\rho=\lambda^*-\bar{\eta}(1-\frac{\delta}{T})>0$,

where $\beta_1=\varepsilon^{-1}\varepsilon_2\lambda_{\max}(L^2)$, $\beta_2=\varepsilon^{-1}\mu_2\lambda_{\max}(L^2)$, γ and η are the solutions of equations $\alpha_1-\gamma-\beta_1e^{\gamma\tau}=0$ and $\alpha_2+\beta_2e^{\eta\tau}=\eta$ respectively, $\bar{\gamma}=\max\{\beta_1,\beta_2\}$, λ^* are the solutions of equation $\lambda^*-\alpha_1+\bar{\gamma}\exp\{\lambda^*\tau\}=0$.

Proof Constructing Lyapunov function:

$$V(t)=e^T(t)e(t)+z^T(t)z(t).$$

When $t\in[-\tau,0]$, let $V_\tau=\sup\limits_{-\tau\le k\le 0}V(k)$, and when $t=0$, we have $V_0\le V_\tau$. Then the derivative of $V(t)$ about t is as follows:

(1) In the control interval, i.e., $t\in(nT,nT+\delta]$, from lemma 1 and lemma 2, we can infer

$$\begin{aligned}\dot{V}(t)&=2e^T(t)\dot{e}(t)+2z^T(t)\dot{z}(t)\\&=\frac{1}{\varepsilon}2e^T(t)(-Ae(t)+Dg(e(t))+D_\tau g(e(t-\tau))+Bz(t)-Q_1e(t))\ +2z^T(t)(-Cz(t)+g(e(t)))-Q_2z(t))\\&\le\frac{1}{\varepsilon}e^T(t)(-2A-2Q_1+\varepsilon_1^{-1}D^TD+\varepsilon_1L^2+\varepsilon_2^{-1}D_\tau^TD_\tau+\varepsilon\varepsilon_3L^2+\varepsilon_4^{-1}I)e(t)\\&\quad+z^T(t)(-2C-2Q_2+\varepsilon^{-1}\varepsilon_3^{-1}I+\varepsilon_4B^TB)z(t)+\varepsilon^{-1}\varepsilon_2e^T(t-\tau)L^2e(t-\tau)\\&\le-\alpha_1V(t)+\beta_1V(t-\tau)+\frac{1}{\varepsilon}e^T(t)(-2A-2Q_1+\varepsilon_1^{-1}D^TD+\varepsilon_1L^2\ +\varepsilon_2^{-1}D_\tau^TD_\tau\\&\quad+\varepsilon\varepsilon_3L^2+\varepsilon_4^{-1}I+\varepsilon\alpha_1I)e(t)+z^T(t)(-2C-2Q_2+\varepsilon^{-1}\varepsilon_3^{-1}I\ +\varepsilon^{-1}\varepsilon_4B^TB+\alpha_1I)z(t)\\&\le-\alpha_1V(t)+\beta_1V(t-\tau),\end{aligned}$$

where $\beta_1=\varepsilon^{-1}\varepsilon_2\lambda_{\max}(L^2)$.

(2) In the non-controlling interval, i.e., $t\in(nT+\delta,(n+1)T]$, we have

$$
\begin{aligned}
\dot{V}(t) &= 2e^T(t)\dot{e}(t) + 2z^T(t)\dot{z}(t) \\
&= 2e^T(t)(-Ae(t) + Dg(e(t)) + D_\tau g(e(t-\tau)) + Bz(t)) + 2z^T(t)(-Cz(t) + g(e(t)))) \\
&\le \frac{1}{\varepsilon} e^T(t)(-2A + \mu_1^{-1}D^TD + \mu_1 L^2 + \mu_2^{-1}D_\tau^T D_\tau + \varepsilon\mu_3 L^2 + \mu_4^{-1}I)e(t) \\
&\quad + z^T(t)(-2C + \varepsilon^{-1}\mu_3^{-1}I + \varepsilon^{-1}\mu_4 B^TB)z(t) + \varepsilon^{-1}\mu_2 e^T(t-\tau)L^2 e(t-\tau) \\
&\le \frac{1}{\varepsilon}\alpha_2 V(t) + \beta_2 V(t-\tau) + e^T(t)(-2A + \mu_1^{-1}D^TD + \mu_1 L^2 + \mu_2^{-1}D_\tau^T D_\tau) \\
&\quad + \varepsilon\mu_3 L^2 + \mu_4^{-1}I - \varepsilon\alpha_2 I)e(t) + z^T(t)(-2C + \varepsilon^{-1}\mu_3^{-1}I + \varepsilon^{-1}\mu_4 B^TB - \alpha_2 I)z(t) \\
&\le \alpha_2 V(t) + \beta_2 V(t-\tau),
\end{aligned}
$$

where $\beta_2 = \varepsilon^{-1}\mu_2\lambda_{\max}(L^2)$. Hence, from (1) and (2) we obtain

$$
\begin{cases}
\dot{V}(t) \le -\alpha_1 V(t) + \beta_1 V(t-\tau), & nT < t \le nT + \delta. \\
\dot{V}(t) \le \alpha_2 V(t) + \beta_2 V(t-\tau), & nT + \delta < t \le (n+1)T.
\end{cases} \tag{18}
$$

By lemma 3 and lemma 4, it is not difficult to find out that for any positive integer n

$$
\begin{cases}
V(t) \le V_{nT} e^{-\gamma(t-nT)}, & nT < t \le nT + \delta, \\
V(t) \le V_{nT+\delta} e^{\eta(t-nT-\delta+\tau)}, & nT + \delta < t \le (n+1)T,
\end{cases} \tag{19}
$$

where γ and η are the solutions of equations $\alpha_1 - \gamma - \beta_1 e^{\gamma\tau} = 0$ and $\alpha_2 + \beta_2 e^{\eta\tau} = \eta$ respectively. Obviously, $V(t)$is an non-increasing function in $t \in (nT \le t \le nT + \delta]$, but it is non-decreasing function in $t \in (nT + \delta \le t \le (n+1)T]$. $V_{nT} = \sup\limits_{nT-\tau \le t \le nT} V(t)$ at $t = nT$ and $V_{nT+\delta} = \sup\limits_{nT+\delta-\tau \le t \le nT+\delta} V(t)$ at $t = nT + \delta$. Since there is no special limit on the scale size of delay τ, it is possible that $nT - \tau$ belongs to interval $((n-1)T, (n-1)T + \delta]$, or to interval $((n-1)T + \delta, nT]$. Similarly, it is also possible that $nT + \delta - \tau$ belongs to interval $((n-1)T + \delta, nT]$ or $(nT, nT + \delta]$. Therefore, the following proof process needs to be divided into two cases:

Case 1: When $\delta - \tau > 0$, we need to prove

$$
\begin{cases}
V(t) \le V_\tau e^{-[\gamma t - n\gamma(T-\delta+\tau) - n(T-\delta+\tau)]}, & t \in (nT, nT + \delta], \\
V(t) \le V_\tau e^{-\{(n+1)[\gamma(\delta-\tau) - \eta(T-\delta+\tau)] - \eta[t-(n+1)T]\}}, & t \in (nT + \delta, (n+1)T], \\
V_{nT} \le V_\tau e^{-n[\gamma(\delta-\tau) - \eta(T-\delta+\tau)]}, & \\
V_{nT+\delta} \le V_\tau e^{-[(n+1)\gamma(\delta-\tau) - n\eta(T-\delta+\tau)]}. &
\end{cases} \tag{20}
$$

From assuming 2 , there exists $V_\tau \ge V_0$.

The following is proved by mathematical induction.

(i) When $l = 0$, i.e., when $t \in [0, T]$, by (18), we have

$$
\begin{cases}
V(t) \le V_0 e^{-\gamma t} \le V_\tau e^{-\gamma t}, & 0 < t \le \delta, \\
V(t) \le V_\delta e^{\eta(t-\delta+\tau)}, & \delta < t \le T.
\end{cases} \tag{21}
$$

By the above inequality, $V_\delta = \sup\limits_{\delta-\tau \le t \le \delta} V(t) \le V(\delta - \tau) \le V_\tau e^{-\gamma(\delta-\tau)}$, then $t \in [\delta, T]$, $V(t) \le V_\tau e^{-\{[\gamma(\delta-\tau) - \eta(T-\delta+\upsilon)] - \eta(t-T)\}}$. Therefore we have

$$
\begin{cases}
V(t) \le V_\tau e^{-\gamma t}, & 0 < t \le \delta, \\
V(t) \le V_\tau e^{-\{[\gamma(\delta-\tau) - \eta(T-\delta+\tau)] - \eta(t-T)\}}, & \delta < t \le T.
\end{cases} \tag{22}
$$

(ii) Suppose that when $l = n - 1$, the following inequalities hold

$$
\begin{cases}
V(t) \le V_\tau e^{-[\gamma t - (n-1)\gamma(T-\delta+\tau) - (n-1)(T-\delta+\tau)]}, & t \in ((n-1)T, (n-1)T + \delta], \\
V(t) \le V_\tau e^{-\{n[\gamma(\delta-\tau) - \eta(T-\delta+\tau)] - \eta[t-nT]\}}, & t \in ((n-1)T + \delta, nT], \\
V_{(n-1)T} \le V_\tau e^{-(n-1)[\gamma(\delta-\tau) - \eta(T-\delta+\tau)]}, & \\
V_{(n-1)T+\delta} \le V_\tau e^{-[n\gamma(\delta-\tau) - (n-1)\eta(T-\delta+\tau)]}. &
\end{cases} \tag{23}
$$

By deduction, when $l = n$, (23) holds.

When $l = n$, $t \in (nT, (n+1)T]$, by (18), we have

$$\begin{cases} V(t) \le V_{nT} e^{-\gamma t(t-nT)}, & nT \le t \le nT+\delta, \\ V(t) \le V_{nT+\delta} e^{\eta(t-nT+\delta-\tau)}, & nT+\delta < t \le (n+1)T. \end{cases} \tag{24}$$

As long as the expressions V_{nT} and $V_{nT+\delta}$ are derived, the above inequality can be obtained. From lemma 4, we obtain

$$V_{nT} = \sup_{nT-\tau \le t \le nT} V(t) = \max\{\sup_{nT-\tau \le t \le (n-1)T+\delta} V(t), \sup_{(n-1)T+\delta \le t \le nT} V(t)\} = \max\{V(nT-\tau), V(nT)\}.$$

because $(n-1)T < nT-\tau \le (n-1)T+\delta$, and by (24) we have $V_{nT} \le V_\tau e^{-n[\gamma(\delta-\tau)-\eta(T-\delta+\tau)]}$. In addition, $V_{nT+\delta} = \sup_{nT+\delta-\tau \le t \le nT+\delta} V(t) \le V(nT+\delta-\tau)$. By (23) we have $V_{nT+\delta} \le V_{nT} e^{-\gamma(\delta-\tau)} \le V_\tau e^{-[(n+1)\gamma(\delta-\tau)-n\eta(T-\delta+\tau)]}$.

From the above analysis process, the following results are obtained. For any $t \ge 0$, there exists $n_0 \in Z^+$, let $\frac{t-\delta}{T} \le n_0 \le \frac{t}{T}$, $t \in [n_0 T, n_0 T+\delta)$ in Case 1, i.e., $\delta - \tau > 0$, we have

$$\begin{aligned} V(t) &\le V_\tau e^{-[\gamma n_0 T - n_0 \gamma(T-\delta+\tau) - n_0 \eta(T-\delta+\tau)]} \\ &\le V_\tau e^{-[\frac{t-\delta}{T}\gamma T - \frac{t}{T}\gamma(T-\delta+\tau) - \frac{t}{T}\eta(T-\delta+\tau)]} \\ &= V_\tau e^{-[\frac{t}{T}(\gamma+\eta)(\delta-\tau) - \eta t - \gamma\delta]}, \end{aligned} \tag{25}$$

and

$$\begin{aligned} V(t) &\le V_\tau e^{-[(n_0+1)\gamma(\delta-\tau) - \eta(T-\delta+\tau)]} \\ &\le V_\tau e^{-[\frac{t}{T}(\gamma(\delta-\tau)-\eta(T-\delta+\tau)]}. \end{aligned} \tag{26}$$

Based on (25) and (26), it can be concluded that $V(t) \le V_\tau e^{(\frac{t-\delta}{T}+1)\eta(T-\delta+\tau)}$. So, for $t > 0$, $\|e^T(t)e(t) + z^T(t)z(t)\| \le \|e^T(t)e(t)\|^2 + \|z^T(t)z(t)\|^2 \le V_\tau e^{-[\frac{t}{T}(\gamma(\delta-\tau)-\eta(T-\delta+\tau)]}$. According to the condition (5)(i), we have $\lim_{t\to+\infty} \|e(t)\| = 0$, $\lim_{t\to+\infty} \|z(t)\| = 0$. Hence the error system is globally exponentially stable.

Case 2: When $0 < \delta < \tau < T$, according to the condition (5)(ii), we can get directly from lemma 5 $V(t) \le V_\tau e^{-\rho t}$. Therefore, $\lim_{t\to+\infty} \|e(t)\| = 0$, $\lim_{t\to+\infty} \|z(t)\| = 0$

Remark 2 In [17], the authors only considered the case of $\tau < \delta$(small delay), and gave some intermittent control synchronization criteria. In the process of proving the theorem 1, according to the relationship between τ and δ, we mainly divide it into two parts to prove the conclusion of the theorem. For the case of $\tau < \delta$(small delay), we use lemma 3 and 4 to get the condition of synchronization. In the case of $\tau \ge \delta$, we can not get the relevant conclusion directly from lemma 3 and 4. Therefore, lemma 5 is given. According to lemma 5, we can directly obtain the criterion of synchronization. Obviously, the conclusion is more general than that in [17].

Remark 3 In this paper, we choose the controllers $u_1(t)$ and $u_2(t)$ as linear batch controllers. In fact, there are similar results for nonlinear batch controllers. For example, we select

$$u_1(t) = \begin{cases} -Q(e(t)), & t \in (nT, nT+\delta], \\ 0, & t \in (nT+\delta, (n+1)T], \end{cases}$$

$$u_2(t) = \begin{cases} -Q(z(t)), & t \in (nT, nT+\delta], \\ 0, & t \in (nT+\delta, (n+1)T]. \end{cases}$$

And we only need to assume that the continuous function $Q(\cdot)$ satisfies the following condition

$$e^T(t)Q(e(t)) \le e^T(t)Q_1 e(t), \quad z^T(t)Q(z(t)) \le z^T(t)Q_2 z(t).$$

In particular, if there is no delay in the neuron transfer function of the model (3), then the model (3) degenerates to the following form

$$\begin{cases} STM: \varepsilon\dot{x}(t) = -Ax(t) + Df(x(t)) + BS(t), \\ LTM: \dot{S}(t) = -CS(t) + f(x(t)). \end{cases} \tag{27}$$

Therefore, the system is described as follows

$$\begin{cases} STM: \varepsilon\dot{y}(t) = -Ay(t) + Df(y(t)) + BR(t) + u_1(t), \\ LTM: \dot{R}(t) = -CR(t) + f(y(t)) + u_2(t), \end{cases} \tag{28}$$

Where, $u_i(t)$, $i = 1, 2$ is an intermittent controller

$$u_1(t) = \begin{cases} -Q_1 e(t), & nT \le t < nT + \delta, \\ 0, & nT + \delta \le t < (n+1)T, \end{cases} \tag{29}$$

$$u_2(t) = \begin{cases} -Q_2 z(t), & nT \le t < nT + \delta, \\ 0, & nT + \delta \le t < (n+1)T, \end{cases} \tag{30}$$

$Q_i \in R^{N\times N}$, $i = 1, 2$ is the gain matrix of feedback control. Let $e(t) = y(t) - x(t)$, $z(t) = R(t) - S(t)$ then the error dynamic system can be written as

$$\begin{cases} STM: \varepsilon\dot{e}(t) = -Ae(t) + Dg(e(t)) + Bz(t) + u_1(t), \\ LTM: \dot{z}(t) = -Cz(t) + g(e(t)) + u_2(t), \end{cases} \tag{31}$$

where $g(e(t)) = f(y(t)) - f(x(t))$.

Corollary 1 Under assumption 1, the system (27) and (28) are synchronized with the intermittent control protocol, if there exists positive numbers α_1, α_2, ε_i, $\mu_i (i = 1, 2, 3)$ and positive definite matrices Q_1, Q_2 meeting the following conditions

(1)$-2A - 2Q_1 + \varepsilon_1^{-1} D^T D + \varepsilon_1 L^2 + \varepsilon\varepsilon_2 L^2 + \varepsilon_3^{-1} I + \varepsilon\alpha_1 I \le 0$,

(2)$-2C - 2Q_2 + \varepsilon^{-1}\varepsilon_2^{-1} I + \varepsilon^{-1}\varepsilon_3 B^T B + \alpha_1 I \le 0$,

(3)$-2A + \mu_1^{-1} D^T D + \mu_1 L^2 + \varepsilon\mu_2 L^2 + \mu_3^{-1} I - \varepsilon\alpha_2 I \le 0$,

(4)$-2C + \varepsilon^{-1}\mu_2^{-1} I + \varepsilon^{-1}\mu_3 B^T B - \alpha_2 I \le 0$,

(5)$\alpha_1\delta - \alpha_2(T - \delta) > 0$.

Proof Constructing Lyapunov function $V(t) = e^T(t)e(t) + z^T(t)z(t)$, when $t \in [nT, nT + \delta)$, by the derivative of $V(t)$, we get

$$\begin{aligned} \dot{V}(t) &= 2e^T(t)\dot{e}(t) + 2z^T(t)\dot{z}(t) \\ &= \frac{2}{\varepsilon} e^T(t)(-Ae(t) + Dg(e(t)) + Bz(t) - Q_1 e(t)) \ + 2z^T(t)(-Cz(t) + g(e(t))) - Q_2 z(t)) \\ &\le \frac{1}{\varepsilon} e^T(t)(-2A - 2Q_1 + \varepsilon_1^{-1} D^T D + \varepsilon_1 L^2 + \varepsilon\varepsilon_2 L^2 + \varepsilon_3^{-1} I)e(t) \\ &\quad + z^T(t)(-2C - 2Q_2 + \varepsilon^{-1}\varepsilon_2^{-1} I + \varepsilon_3 B^T B)z(t) + \varepsilon^{-1}\varepsilon_2 e^T(t-\tau) L^2 e(t-\tau) \\ &\le -\alpha_1 V(t) + \frac{1}{\varepsilon} e^T(t)(-2A - 2Q_1 + \varepsilon_1^{-1} D^T D + \varepsilon_1 L^2 + \varepsilon\varepsilon_2 L^2 + \varepsilon_3^{-1} I \\ &\quad + \varepsilon\alpha_1 I)e(t) + z^T(t)(-2C - 2Q_2 + \varepsilon^{-1}\varepsilon_2^{-1} I + \varepsilon^{-1}\varepsilon_3 B^T B + \alpha_1 I)z(t) \\ &\le -\alpha_1 V(t). \end{aligned}$$

Hence, $V(t) \le V(nT) e^{-\alpha_1 (t - nT)}$. When $t \in [nT + \delta, (n+1)T)$, we have

$$\begin{aligned} \dot{V}(t) &= 2e^T(t)\dot{e}(t) + 2z^T(t)\dot{z}(t) \\ &= 2e^T(t)(-Ae(t) + Dg(e(t)) + Bz(t)) + 2z^T(t)(-Cz(t) + g(e(t)))) \\ &\le \frac{1}{\varepsilon} e^T(t)(-2A + \mu_1^{-1} D^T D + \mu_1 L^2 + + \varepsilon\mu_2 L^2 + \mu_3^{-1} I)e(t) \ + z^T(t)(-2C + \varepsilon^{-1}\mu_2^{-1} I + \varepsilon^{-1}\mu_3 B^T B)z(t) \\ &\le \alpha_2 V(t) + e^T(t)(-2A + \mu_1^{-1} D^T D + \mu_1 L^2 + \varepsilon\mu_2 L^2 + \mu_3^{-1} I - \varepsilon\alpha_2 I)e(t) + z^T(t)(-2C + \varepsilon^{-1}\mu_2^{-1} I + \varepsilon^{-1}\mu_3 B^T B - \alpha_2 I)z(t) \\ &\le \alpha_2 V(t). \end{aligned}$$

then

$$V(t) \le V(nT + \delta) e^{\alpha_2 (t - (nT + \delta))} \le V(nT) e^{-\alpha_1\delta + \alpha_2 (t - (nT + \delta))}.$$

So $V((n+1)T) \le V(nT) e^{-\alpha_1\delta + \alpha_2 (T - \delta)}$. For $\forall t \in [nT, nT + \delta)$, by the mathematical induction, we can obtain obviously

$$V(t) \le V(t_0)e^{-n[\alpha_1\delta-\alpha_2(T-\delta)}e^{-\alpha_1(t-nT)} \le V(t_0)e^{-(\alpha_1\delta-\alpha_2(T-\delta))\frac{t-\delta}{T}}.$$

For $\forall t \in [nT+\delta,(n+1)T)$,

$$V(t) \le V(t_0)e^{-(\alpha_1\delta-\alpha_2(T-\delta))\frac{t-\delta}{T}} \cdot e^{-(\alpha_1\delta-\alpha_2(T-\delta))} = V(t_0)e^{-(\alpha_1\delta-\alpha_2(T-\delta))(\frac{t-\delta}{T}+1)}.$$

By the condition (5) of inference 1, we have $\lim_{t\to+\infty} V(t)=0$, and then

$$\lim_{t\to+\infty} \|e(t)\|=0, \quad \lim_{t\to+\infty} \|z(t)\|=0.$$

Remark 4 In this section, we require $0<\delta<T$. In particular, when $\delta=t$, the controllers (5) and (6) degenerate into general continuous control, i.e.,

$$u_1(t)=-Q_1e(t), \quad u_2(t)=-Q_2z(t). \tag{32}$$

Corollary 2 Under assumings 1, 2 , the system (3) and system (4) are synchronized with the controller (32), if there exist positive numbers $\alpha_1,\varepsilon_i(i=1,\cdots,4)$ and positive definite matrices Q_1, Q_2 meeting the following conditions

(1)$-2A-2Q_1+\varepsilon_1^{-1}D^TD+\varepsilon_1L^2+\varepsilon_2^{-1}D_\tau^TD_\tau+\varepsilon\varepsilon_3L^2+\varepsilon_4^{-1}I+\varepsilon\alpha_1I\le 0$,

(2)$-2C-2Q_2+\varepsilon^{-1}\varepsilon_3^{-1}I+\varepsilon^{-1}\varepsilon_4B^TB+\alpha_1I\le 0$.

3 Numerical example and simulation

Consider a numerical example of $2-D$ competitive neural networks with variable time delay

$$\begin{cases} STM: \varepsilon\dot{x}(t)=-Ax(t)+Df(x(t))+D_\tau f((t-\tau))+BS(t), \\ LTM: \dot{S}(t)=-CS(t)+f(x(t)), \end{cases} \tag{33}$$

where $x(t)=(x_1(t),x_2(t))^T$, $S(t)=(s_1(t),s_2(t))^T$, $f(x)=(0.6\tanh(x_1)+0.4\sin(x_1), 0.6\tanh(x_2)+0.4\sin(x_2))^T$, $\tau=0.4$, $C=0.8I$, $B=0.3I$, $\varepsilon=0.4$,

$$A=\begin{pmatrix}1.5 & 0\\ 0 & 2.2\end{pmatrix},\ D=\begin{pmatrix}1 & -1.8\\ 1 & 2.5\end{pmatrix},\ D_\tau=\begin{pmatrix}-1.45 & -0.05\\ -0.28 & 0.05\end{pmatrix}.$$

The initial conditions of this dynamic behavior system are $x(0)=(2,0.4)^T$, $S(0)=(-0.5,0.8)^T$, and the operation results are shown in Fig 1, Fig 2.

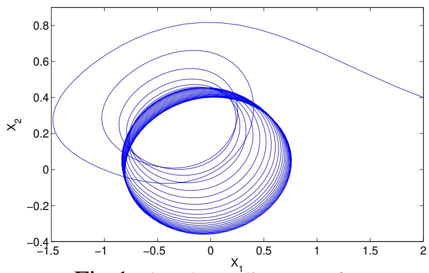

Fig 1 the phase diagram of $x(t)$

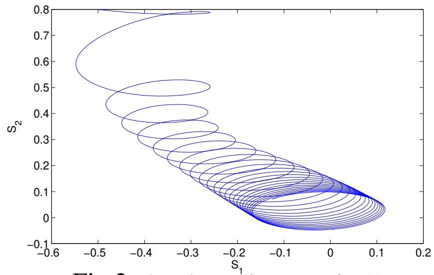

Fig 2 the phase diagram of $S(t)$

Consider the synchronization between the main system (33) and the slave systems (34)

$$\begin{cases} STM: \varepsilon\dot{y}(t)=-Ay(t)+Df(y(t))+D_\tau f((t-\tau))+BR(t)+u_1(t), \\ LTM: \dot{R}(t)=-CR(t)+f(y(t))+u_2(t), \end{cases} \tag{34}$$

where $y(t)=(y_1(t),y_2(t))^T$, $R(t)=(r_1(t),r_2(t))^T$. The state with initial conditions $y(0)=(-1.6,0.6)^T$, $R(0)=(0.2,0.56)^T$, $x_i(t)$, $y_i(t)$, $s_i(t)$ and $r_i(t)(i=1,2)$ have no control, as shown in Fig 3~Fig 6. Obviously, the trajectory of master-slave system is not synchronous without control.

According to the analysis of theorem 1, the main system (33) and the slave system (34) tend to be synchronized under appropriate conditions after periodic intermittence control (5) and (6) are added to the system. For convenience, we choose $Q_1=3.5I$, $Q_1=3I$, $\varepsilon_1=\mu_1=2$, $\varepsilon_2=\mu_2=1$, $\varepsilon_3=\mu_3=0.5$, $\varepsilon_4=\mu_4=0.25$. By simple calculation, it is found that the conditions (1)~(4) of theorem 1 hold. As long as we choose $\delta=2$, $T=3$, the condition (5)(i) of theorem 1 holds. So, in this example, where $\delta=2$, $T=3$, under the periodic intermittent control (5) and (6), track patterns of $x_i(t)$, $y_i(t)$, $s_i(t)$, and $r_i(t)(i=1,2)$ are shown in Fig 7~Fig 10. Obviously, the master-slave system tends to be synchronized.

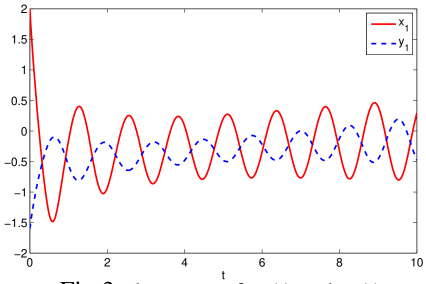

Fig 3 the states of $x_1(t)$ and $y_1(t)$

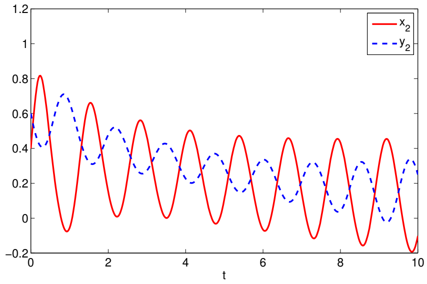

Fig 4 the states of $x_2(t)$ and $y_2(t)$

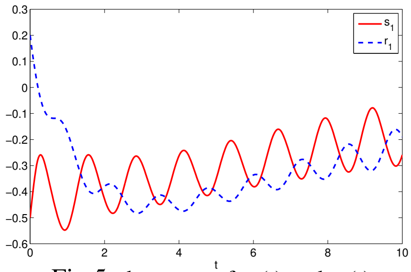

Fig 5 the states of $s_1(t)$ and $r_1(t)$

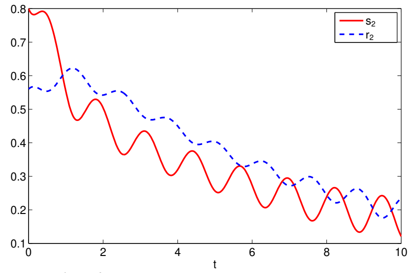

Fig 6 the states of $s_2(t)$ and $r_2(t)$

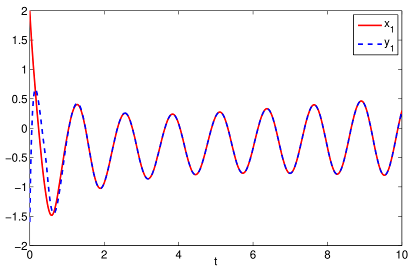

Fig 7 the states of $x_1(t)$ and $y_1(t)$

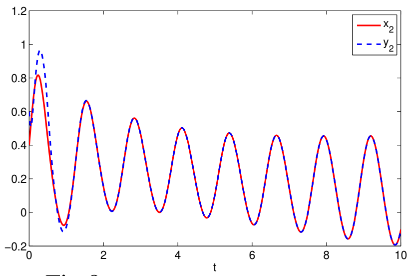

Fig 8 the states of $x_2(t)$ and $y_2(t)$

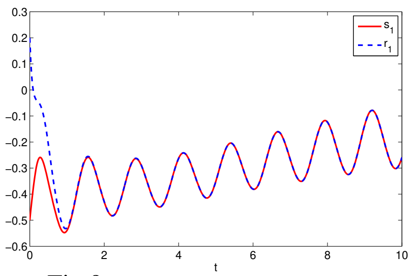

Fig 9 the states of $s_1(t)$ and $r_1(t)$

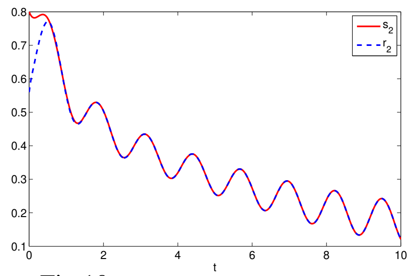

Fig 10 the states of $s_2(t)$ and $r_2(t)$

4 Conclution

This paper considers a competitive neural networks model with time-varying delay based on the neural networks with time-varying delay in which the intermittent feedback control with non-zero duration can be stabilized to continuous time. By using the method of differential inequality analysis and Lyapunov function, the global exponential synchronization conditions of competitive neural networks with time delay are discussed, and the corresponding criteria are established. At the same time, the exponential synchronization problems of competitive neural networks with time-varying delays and intermittent control are studied, and which satisfied both large and small delays. Some sufficient conditions are established. Finally, a simulation example is shown to verify the correctness and validity of the theoretical results.

间歇控制下时滞竞争神经网络指数同步探究*

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摘 要： 本文主要讨论了间歇控制下时滞竞争神经网络的指数同步问题. 利用微分不等式分析方法与 Lyapunov 函数法，给出了同时满足大时滞与小时滞竞争神经网络的全局指数同步的充分性条件，并建立了相应的判别准则. 最后，设计了一个数值实验，通过仿真实例验证了所提出理论的正确性和有效性.

关键词： 竞争神经网络; 同步; 时变时滞; 间歇控制

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0 引 言

竞争神经网络模型是一类重要的无监督突触修饰神经网络, 它包含两种状态变量, 短期记忆变量(STM)描述快速神经活动, 长期记忆变量(LTM)描述缓慢的神经活动. 自1983年Cohen和Grossberg[1]首次提出了竞争神经网络以来, 出现了很多关于竞争神经网络的研究, 如不同时间尺度竞争神经网络[2−5]、时变分布时滞竞争神经网络[6,7]、无监督竞争神经网络[8]、不同时间尺度和随机扰动下竞争神经网络[9]、具有不连续激活函数的时滞竞争神经网络[10]、具有不同时间尺度和时变延迟的竞争神经网络[11].

此外, 同步是一种典型的集体行为, 它在生物进化、化学反应、安全通信等方面应用广泛. 1990年, 其开创性的研究工作由Pecora和Carroll[12]提出, 自此竞争神经网络的同步问题受到了越来越多学者的关注. Lou与Cui[4]研究了一类竞争神经网络的指数同步问题, 利用线性矩阵不等式方法、Lyapunov函数法、牛顿-莱布尼兹公式, 设计了一些指数同步准则. Gu[9]设计了一种自适应反馈控制器, 以实现具有不同时间尺度和随机扰动的耦合时滞竞争神经网络的完全同步，利用随机微分时滞方程的Lasalle不变性原理, 研究了误差动态系统的全局几乎确定的渐近稳定性. Gan[11]研究了一类具有不同尺度时变时滞的竞争神经网络的同步问题，提出了一种新的延迟划分方法, 推导了一种保证响应系统与驱动系统同步的延迟相关条件，通过求解线性矩阵不等式, 可以实现线性反馈控制器增益矩阵的设计. Zhang等人[13]研究了混合脉冲和切换控制下复杂动态网络的同步问题. 研究表明, 许多大型复杂动力网络都表现出集体同步运动, 可利用脉冲控制的概念和脉冲系统的稳定结果, 建立该模型的脉冲同步新准则.

近年来, 间歇控制等非连续反馈控制策略在交通、通信等工程中得到了广泛的应用[14−16]. 因此, 对于间歇控制方法的研究十分必要, 如文献[17]采用周期性间歇非线性反馈控制方法, 对一类具有时滞的非线性混沌系统进行同步，基于Halanay不等式和Lyapunov泛函理论等方法, 推导出了一些同步准则, 并得到了保证非线性系统无延迟同步的充分条件. 文献[18]用周期间歇控制方法研究了一类非线性系统的指数稳定问题，给出了一组线性矩阵不等式的指数稳定判据, 以及由三个标量不等式确定的一个简单的充分条件. 在控制周期和控制宽度固定且已知的前提下, 针对一般成本函数设计了次优间歇控制器, 并用数值模拟验证了理论结果. 文献[19]采用周期间歇控制方法研究了延迟混沌神经网络的指数稳定问题，利用Lyapunov函数和Halanay不等式, 建立了受控神经网络的指数稳定准则及其简化形式, 对控制参数的可行域进行了严格的估计. 理论结果和数值模拟表明, 采用非零持续时间的间歇反馈控制可以稳定连续时间的时滞混沌神经网络.

本文在以上研究的基础上考虑了具有时变时滞的竞争神经网络模型, 利用微分不等式分析方法和Lyapunov

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函数方法, 讨论了具有时滞的竞争神经网络的全局指数同步条件, 并建立了相应的判别准则, 同时, 也研究了既满足大时滞也满足小时滞的时变时滞和间歇控制的竞争神经网络的指数同步问题, 并建立了一些充分性条件. 最后, 通过仿真实例验证了所提出方案的正确性和有效性.

1 模型建立及预备知识

考虑如下具有时滞的竞争神经网络

$$\begin{cases} STM:\varepsilon\dot{x}_i(t)=-a_ix_i(t)+\sum\limits_{k=1}^{N}d_{ik}f_k(x_k(t))+\sum\limits_{k=1}^{N}d_{ik}^{\tau}f_k(x_k(t-\tau))+b_i\sum\limits_{j=1}^{P}m_{ij}(t)\sigma_j, \\ LTM:\dot{m}_{ij}(t)=-c_im_{ij}(t)+\sigma_jf_i(x_i(t)), \end{cases} \tag{1}$$

其中: P为受到外部刺激的神经元个数, N为STM状态下的联接神经元个数, x_i表示神经元当前被激活水平, $f_i(\cdot)$表示神经元输出函数, $m_{ij}(t)$记为有效联接, $a_i>0$为神经元激活常数, σ_j记为外部刺激权重, b_i代表外部刺激程度, $c_i>0$为任意常数, d_{ik}代表第i个和第k个神经元之间的联接权重, d_{ik}^{τ}代表第i个和第k个神经元之间时滞反馈的联接权重, τ为时滞常数, $\varepsilon>0$表示STM状态下的时间尺度.

令$s_i(t)=\sum\limits_{j=1}^{P}m_{ij}(t)\sigma_j=m_i(t)\sigma$, 其中: $\sigma=(\sigma_1,\sigma_2,\cdots,\sigma_P)^T$, $m_i(t)=(m_{i1}(t),m_{i2}(t),\cdots,m_{iP}(t))^T$, 则系统(1)可被表示如下

$$\begin{cases} STM:\varepsilon\dot{x}_i(t)=-a_ix_i(t)+\sum\limits_{k=1}^{N}d_{ik}f_k(x_k(t))+\sum\limits_{k=1}^{N}d_{ik}^{\tau}f_k(x_k(t-\tau))+b_is_i(t), \\ LTM:\dot{s}_i(t)=-c_is_i(t)+|\sigma|^2f_i(x_i(t)), \end{cases} \tag{2}$$

其中: $|\sigma|^2=\sigma_1^2+\sigma_1^2+\cdots+\sigma_P^2$为一常数, 不失一般性, 不妨令$|\sigma|^2=1$, 且在$t\in\mathbb{R}^+=[0,+\infty)$时, 令$x(t)=(x_1(t),x_2(t),\cdots,x_N(t))^T$, $S(t)=(s_1(t),s_2(t),\cdots,s_N(t))^T$, $A=\mathrm{diag}(a_1,a_2,\cdots,a_N)$, $D=(d_{ik})_{N\times N}$, $D_\tau=(d_{ik}^{\tau})_{N\times N}$, $B=\mathrm{diag}(b_1,b_2,\cdots,b_N)$, $C=\mathrm{diag}(c_1,c_2,\cdots,c_N)$, $f(x(t))=(f_1(x_1(t)),f_2(x_2(t)),\cdots,f_N(x_N(t)))^T$, $f(x(t-\tau))=(f_1(x_1(t-\tau)),f_2(x_2(t-\tau)),\cdots,f_N(x_N(t-\tau)))^T$.
于是把系统(2)写成矩阵形式为

$$\begin{cases} STM:\varepsilon\dot{x}(t)=-Ax(t)+Df(x(t))+D_\tau f(x(t-\tau))+BS(t), \\ LTM:\dot{S}(t)=-CS(t)+f(x(t)). \end{cases} \tag{3}$$

因此，从系统描述如下

$$\begin{cases} STM:\varepsilon\dot{y}(t)=-Ay(t)+Df(y(t))+D_\tau f(y(t-\tau))+BR(t)+u_1(t), \\ LTM:\dot{R}(t)=-CR(t)+f(y(t))+u_2(t), \end{cases} \tag{4}$$

其中: $y(t)=(y_1(t),y_2(t),\cdots,y_N(t))^T$, $R(t)=(r_1(t),r_2(t),\cdots,r_N(t))^T$, $u_i(t)(i=1,2)$为间歇控制器

$$u_1(t)=\begin{cases} -Q_1e(t), & nT\le t<nT+\delta, \\ 0, & nT+\delta\le t<(n+1)T, \end{cases} \tag{5}$$

$$u_2(t)=\begin{cases} -Q_2z(t), & nT\le t<nT+\delta, \\ 0, & nT+\delta\le t<(n+1)T, \end{cases} \tag{6}$$

$Q_i\in R^{N\times N}(i=1,2)$为反馈控制增益矩阵. 令$e(t)=y(t)-x(t)$, $z(t)=R(t)-S(t)$, 则误差动力系统可写为

$$\begin{cases} STM:\varepsilon\dot{e}(t)=-Ae(t)+Dg(e(t))+D_\tau g(e(t-\tau))+Bz(t)+u_1(t), \\ LTM:\dot{z}(t)=-Cz(t)+g(e(t))+u_2(t), \end{cases} \tag{7}$$

其中: $g(e(t))=f(y(t))-f(x(t))$, $g(e(t-\tau))=f(y(t-\tau))-f(x(t-\tau))$.

假设1 存在一对称正定阵$L=(l_{ij})_{m\times n}$, 对任意$x,y\in R^n$，有$\|f(x)-f(y)\|\le L\|x-y\|$.

假设2 时滞$\tau<T$, 其中τ为常数, T为周期.

引理1[18] 对任意具有恰当维度的实矩阵$\Sigma_1,\Sigma_2,\Sigma_3$, 若给定$0<\Sigma_3=\Sigma_3^T$与任意实数$\varepsilon>0$, 则不等式$\Sigma_1^T\Sigma_2+\Sigma_2^T\Sigma_1\leq\varepsilon\Sigma_1^T\Sigma_3\Sigma_1+\varepsilon^{-1}\Sigma_2^T\Sigma_3^{-1}\Sigma_2$成立.

引理2[13] 若$P\in R^{m\times n}$是一正定矩阵, Q是一对称阵, 则有$\lambda_{\min}(P^{-1}Q)X^TPX\leq X^TQX\leq\lambda_{\max}(P^{-1}Q)X^TPX$成立, 其中$X\in R^n$.

引理3[17] 令φ是区间$[t_0-\tau,+\infty)$内的非负函数, 且在子区间$[t_0,+\infty)$内连续. 设$t\geq t_0$时$\varphi'\leq\alpha\varphi(t)+\beta\varphi(t-\tau)$. 若$\alpha>0$, $\beta>0$, $t\geq t_0$时, 有$\varphi(t)\leq\varphi_{t_0}e^{\eta(t-t_0+\tau)}$, 其中$\varphi_{t_0}=\sup_{t_0-\tau\leq\theta\leq t_0}\varphi(\theta)$, 且$\eta>0$为方程$\alpha+\beta e^{-\eta\tau}=\eta$的唯一解.

引理4(Halanay不等式)[19] 令$\varphi:[t_0-\tau)\to[0,+\infty)$是一连续函数, 在$t\geq t_0$时, 有$\varphi'\leq-\alpha\varphi(t)+\beta\varphi_t$. 若$\alpha>0$, $\beta>0$, 那么, $t\geq t_0$ 时, 有$\varphi(t)\leq\varphi_{t_0}e^{-\gamma(t-t_0)}$, 其中$\varphi_t=\sup_{t-\tau\leq\theta\leq t}\varphi(\theta)$, 且$\gamma>0$为方程$\alpha-\gamma-\beta e^{\gamma\tau}=0$的唯一解.

为了给出更一般时滞（即不区分大小时滞）的结论, 我们给出并证明如下引理.

引理5 假设函数$y(t)$在$t\in[-\tau,+\infty)$上是连续的非负函数. 如果存在常数γ_1, γ_2, γ_3, γ_4满足以下条件

$$\begin{cases}\dot{y}(t)\leq-\gamma_1y(t)+\gamma_2y(t-\tau), & t\in[nT,nT+\delta),\\ \dot{y}(t)\leq\gamma_3y(t)+\gamma_4y(t-\tau), & t\in[nT+\delta,(n+1)T),\end{cases}\tag{8}$$

且$\gamma_1>\bar{\gamma}\geq0$, $\bar{\eta}=\gamma_1+\gamma_3>0$, $\rho=\lambda^*-\bar{\eta}(1-\frac{\delta}{T})>0$, 则$y(t)\leq\sup\limits_{-\tau\leq s\leq0}y(s)\exp\{-\rho t\}$, 其中$0<\delta<T$, $\bar{\gamma}=\max\{\gamma_2,\gamma_4\}$, λ^* 是方程$\lambda^*-\gamma_1+\bar{\gamma}\exp\{\lambda^*\tau\}=0$的解.

证明 设$\mathcal{W}(t)=y(t)\exp\{\lambda^*t\}(t\geq0)$, $\mathcal{M}_0=\sup\limits_{-\tau\leq s\leq0}y(s)$, 其中$\lambda^*$是方程$\lambda^*-\gamma_1+\bar{\gamma}\exp\{\lambda^*\tau\}=0$的解. 由引理4, 我们得到$\mathcal{W}(t)\leq\mathcal{M}_0$. 因此, 对于任意的$t\in[0,\delta)$, 我们有$\mathcal{W}(t)<h\mathcal{M}_0$, 其中常数$h>1$. 于是我们需要证明对于任意的$t\in[\delta,T)$, 有$\mathcal{W}(t)<h\mathcal{M}_0\exp\{\bar{\eta}(t-\delta)\}$成立. 即

$$\mathcal{H}(t)=\mathcal{W}(t)-h\mathcal{M}_0\exp\{\bar{\eta}(t-\delta)\}<0.\tag{9}$$

否则, 存在$p_0\in(\delta,T)$使得

$$\mathcal{H}(p_0)=0,\ \dot{\mathcal{H}}(p_0)\geq0,\ \mathcal{H}(t)<0,\ t\in(\delta,p_0).\tag{10}$$

对于$\tau>0$, 如果$\delta<p_0-\tau(p_0)<p_0$, 则$\mathcal{W}(p_0-\tau(p_0))<h\mathcal{M}_0\exp\{\bar{\eta}(p_0-\delta)\}$. 此外, 如果$-\tau\leq p_0-\tau(p_0)\leq\delta$, 我们有$\mathcal{W}(p_0-\tau(p_0))<h\mathcal{M}_0<h\mathcal{M}_0\exp\{\bar{\eta}(p_0-\delta)\}$. 因此, 对任意$\tau$, 有$\mathcal{W}(p_0-\tau(p_0))<h\mathcal{M}_0\exp\{\bar{\eta}(p_0-\delta)\}$. 故

$$\begin{aligned}\dot{\mathcal{H}}(p_0)&=\lambda^*\mathcal{W}(p_0)+\exp\{\lambda^*p_0\}\dot{y}(p_0)-\bar{\eta}h\mathcal{M}_0\exp\{\bar{\eta}(p_0-\delta)\}\\&=(\lambda^*+\gamma_3)\mathcal{W}(p_0)+\gamma_4\exp\{\lambda^*\tau\}\mathcal{W}(p_0-\tau)-\bar{\eta}h\mathcal{M}_0\exp\{\bar{\eta}(p_0-\delta)\}\\&<(\lambda^*+\gamma_3+\gamma_4\exp\{\lambda^*\tau\}-\bar{\eta})h\mathcal{M}_0\exp\{\bar{\eta}(p_0-\delta)\}\\&\leq(\lambda^*-\gamma_1+\bar{\gamma}\exp\{\lambda^*\tau\})h\mathcal{M}_0\exp\{\bar{\eta}(p_0-\delta)\}=0.\end{aligned}\tag{11}$$

这与不等式(10)矛盾. 因此, 不等式(9)成立, 也就是对任意的$t\in[\delta,T)$,

$$\mathcal{W}(t)<h\mathcal{M}_0\exp\{\bar{\eta}(t-\delta)\}<h\mathcal{M}_0\exp\{\bar{\eta}(T-\delta)\}.\tag{12}$$

则对任意的$t\in[-\tau,T)$, 有$\mathcal{W}(t)<h\mathcal{M}_0\exp\{\bar{\eta}(T-\delta)\}$.

下一步证明对于任意的$t\in[T,T+\delta)$, 有

$$\mathcal{W}(t)<h\mathcal{M}_0\exp\{\bar{\eta}(T-\delta)\}.\tag{13}$$

否则, 存在一个$p_1\in[T,T+\delta)$, 使得

$$\tilde{\mathcal{H}}(p_1)=0,\ \dot{\tilde{\mathcal{H}}}(p_1)\geq0,\ \tilde{\mathcal{H}}(t)<0,\ t\in(t_1,p_1),\tag{14}$$

其中: $\tilde{\mathcal{H}}(t)=\mathcal{W}(t)-h\mathcal{M}_0\exp\{\bar{\eta}(T-\delta)\}$. 故

$$\begin{aligned}\dot{\tilde{\mathcal{H}}}(p_1)&=\lambda^*\mathcal{W}(p_1)+\exp\{\lambda^*p_1\}\dot{y}(p_1)\\&=(\lambda^*-\gamma_1)\mathcal{W}(p_1)+\gamma_2\exp\{\lambda^*\tau(p_1)\}\mathcal{W}(p_1-\tau)\\&<(\lambda^*-\gamma_1+\gamma_2\exp\{\lambda^*\tau\})h\mathcal{M}_0\exp\{\bar{\eta}(T-\delta)\}\\&\leq(\lambda^*-\gamma_1+\bar{\gamma}\exp\{\lambda^*\tau\})h\mathcal{M}_0\exp\{\bar{\eta}(T-\delta)\}=0.\end{aligned}\tag{15}$$

这与不等式(14)矛盾. 因此, (13)成立.

对于$t\in[T+\delta,2T)$, 利用相似的方法, 我们得到$\mathcal{W}(t)<h\mathcal{M}_0\exp\{\bar{\eta}[(T-\delta)+(t-(T+\delta))]\}$. 重复上述证明过程, 对于任意的$t_k\le t\le s_k$, 我们有

$$\mathcal{W}(t)<h\mathcal{M}_0\exp\{\bar{\eta}k(T-\delta)\}. \tag{16}$$

此外, 对于$kT+\delta<t<(k+1)T$, 可得

$$\mathcal{W}(t)<h\mathcal{M}_0\exp\{\bar{\eta}[k(T-\delta)+(t-kT+\delta)]\}. \tag{17}$$

由于对任意的$t\ge0$, 存在一个正整数k, 使得$kT\le t\le(k+1)T$. 因此, 对任意的t, 我们对$\mathcal{W}(t)$有如下的估计, 当$kT\le t<kT+\delta$,

$$\begin{aligned}\mathcal{W}(t)&<h\mathcal{M}_0\exp\{\bar{\eta}k(T-\delta)\}\\&=h\mathcal{M}_0\exp\left\{\bar{\eta}k\frac{T-\delta}{T}\cdot T\right\}\\&=h\mathcal{M}_0\exp\left\{\bar{\eta}\left(1-\frac{\delta}{T}\right)kT\right\}\\&\le h\mathcal{M}_0\exp\left\{\bar{\eta}\left(1-\frac{\delta}{T}\right)t\right\}.\end{aligned}$$

当$kT+\delta\le t<(k+1)T$, 有

$$\begin{aligned}\mathcal{W}(t)&<h\mathcal{M}_0\exp\{\bar{\eta}[k(T-\delta)+(t-kT-\delta)]\}\\&=h\mathcal{M}_0\exp\left\{\bar{\eta}\left[k\frac{T-\delta}{T}\cdot T+\frac{t-kT-\delta}{t-kT}\cdot(t-kT)\right]\right\}\\&\le h\mathcal{M}_0\exp\left\{\bar{\eta}\left[k\left(1-\frac{\delta}{T}\right)kT+\frac{\delta}{T}\cdot(t-kT)\right]\right\}\\&\le h\mathcal{M}_0\exp\left\{\bar{\eta}\left(1-\frac{\delta}{T}\right)[kT+(t-kT)]\right\}\\&=h\mathcal{M}_0\exp\left\{\bar{\eta}\left(1-\frac{\delta}{T}\right)t\right\}.\end{aligned}$$

令$h\to1$, 由$\mathcal{W}(t)$的定义, 我们有$y(t)\le\mathcal{M}_0\exp\{-(\lambda^*-\bar{\eta}(1-\frac{\delta}{T}))t\}=\mathcal{M}_0\exp\{-\rho t\}$, $t\ge0$.

注记1 在引理5中, 问题对$\tau<\delta$与$\tau\ge\delta$这两种情况都成立. 也就是说, 此引理中, 我们仅仅要求$\tau<T$, 对于δ和τ之间的大小并没有任何限制.

2 间歇控制下竞争神经网络的指数同步准则

我们基于微分不等式方法和间歇控制策略, 给出了系统(3)与系统(4)同步准则的严格推导.

定理1 若假设1、2成立, 如果存在正常数α_1, α_2, ε_i, $\mu_i(i=1,\cdots,4)$以及正定矩阵Q_1, Q_2满足以下条件

(1)$-2A-2Q_1+\varepsilon_1^{-1}D^TD+\varepsilon_1L^2+\varepsilon_2^{-1}D_\tau^TD_\tau+\varepsilon\varepsilon_3L^2+\varepsilon_4^{-1}I+\varepsilon\alpha_1I\le0$,

(2)$-2C-2Q_2+\varepsilon^{-1}\varepsilon_3^{-1}I+\varepsilon^{-1}\varepsilon_4B^TB+\alpha_1I\le0$,

(3)$-2A+\mu_1^{-1}D^TD+\mu_1L^2+\mu_2^{-1}D_\tau^TD_\tau+\varepsilon\mu_3L^2+\mu_4^{-1}I-\varepsilon\alpha_2I\le0$,

(4)$-2C+\varepsilon^{-1}\mu_3^{-1}I+\varepsilon^{-1}\mu_4B^TB-\alpha_2I\le0$,

(5) (i) 如果$0<\tau<\delta<T$, $\gamma(\delta-\tau)-\eta(T-\delta+\tau)>0$,

(ii) 如果$0<\delta<\tau<T$, $\alpha_1>\bar{\gamma}\ge0$, $\bar{\eta}=\alpha_1+\alpha_2>0$, $\rho=\lambda^*-\bar{\eta}(1-\frac{\delta}{T})>0$,

其中: $\beta_1=\varepsilon^{-1}\varepsilon_2\lambda_{\max}(L^2)$, $\beta_2=\varepsilon^{-1}\mu_2\lambda_{\max}(L^2)$, γ和η分别是方程$\alpha_1-\gamma-\beta_1e^{\gamma\tau}=0$和$\alpha_2+\beta_2e^{\eta\tau}=\eta$的解, $\bar{\gamma}=\max\{\beta_1,\beta_2\}$, λ^*是方程$\lambda^*-\alpha_1+\bar{\gamma}\exp\{\lambda^*\tau\}=0$的解. 那么, 系统(3)和系统(4)在间歇控制协议下实现同步.

证明 构造Lyapunov函数$V(t)=e^T(t)e(t)+z^T(t)z(t)$. 当$t\in[-\tau,0]$时, 记$V_\tau=\sup\limits_{-\tau\le k\le0}V(k)$, 并且当$t=0$时, 有$V_0\le V_\tau$. 那么$V(t)$关于$t$求导如下:

(1) 在控制间歇中, 当$t\in(nT,nT+\delta]$时, 由引理1与引理2可知

$$
\begin{aligned}
\dot V(t)&=2e^T(t)\dot e(t)+2z^T(t)\dot z(t)\\
&=\frac{1}{\varepsilon}2e^T(t)(-Ae(t)+Dg(e(t))+D_\tau g(e(t-\tau))+Bz(t)-Q_1e(t))\\
&\quad+2z^T(t)(-Cz(t)+g(e(t)))-Q_2z(t))\\
&\le\frac{1}{\varepsilon}e^T(t)(-2A-2Q_1+\varepsilon_1^{-1}D^TD+\varepsilon_1L^2+\varepsilon_2^{-1}D_\tau^TD_\tau+\varepsilon\varepsilon_3L^2+\varepsilon_4^{-1}I)e(t)\\
&\quad+z^T(t)(-2C-2Q_2+\varepsilon^{-1}\varepsilon_3^{-1}I+\varepsilon_4B^TB)z(t)+\varepsilon^{-1}\varepsilon_2e^T(t-\tau)L^2e(t-\tau)\\
&\le-\alpha_1V(t)+\beta_1V(t-\tau)+\frac{1}{\varepsilon}e^T(t)(-2A-2Q_1+\varepsilon_1^{-1}D^TD+\varepsilon_1L^2+\varepsilon_2^{-1}D_\tau^TD_\tau\\
&\quad+\varepsilon\varepsilon_3L^2+\varepsilon_4^{-1}I+\varepsilon\alpha_1I)e(t)+z^T(t)(-2C-2Q_2+\varepsilon^{-1}\varepsilon_3^{-1}I+\varepsilon^{-1}\varepsilon_4B^TB+\alpha_1I)z(t)\\
&\le-\alpha_1V(t)+\beta_1V(t-\tau),
\end{aligned}
$$

其中: $\beta_1=\varepsilon^{-1}\varepsilon_2\lambda_{\max}(L^2)$.

(2) 在非控制间歇中, 当$t\in(nT+\delta,(n+1)T]$时, 亦有

$$
\begin{aligned}
\dot V(t)&=2e^T(t)\dot e(t)+2z^T(t)\dot z(t)\\
&=2e^T(t)(-Ae(t)+Dg(e(t))+D_\tau g(e(t-\tau))+Bz(t))+2z^T(t)(-Cz(t)+g(e(t))))\\
&\le\frac{1}{\varepsilon}e^T(t)(-2A+\mu_1^{-1}D^TD+\mu_1L^2+\mu_2^{-1}D_\tau^TD_\tau+\varepsilon\mu_3L^2+\mu_4^{-1}I)e(t)\\
&\quad+z^T(t)(-2C+\varepsilon^{-1}\mu_3^{-1}I+\varepsilon^{-1}\mu_4B^TB)z(t)+\varepsilon^{-1}\mu_2e^T(t-\tau)L^2e(t-\tau)\\
&\le\frac{1}{\varepsilon}\alpha_2V(t)+\beta_2V(t-\tau)+e^T(t)(-2A+\mu_1^{-1}D^TD+\mu_1L^2+\mu_2^{-1}D_\tau^TD_\tau)\\
&\quad+\varepsilon\mu_3L^2+\mu_4^{-1}I-\varepsilon\alpha_2I)e(t)+z^T(t)(-2C+\varepsilon^{-1}\mu_3^{-1}I+\varepsilon^{-1}\mu_4B^TB-\alpha_2I)z(t)\\
&\le\alpha_2V(t)+\beta_2V(t-\tau),
\end{aligned}
$$

其中: $\beta_2=\varepsilon^{-1}\mu_2\lambda_{\max}(L^2)$.

因此, 由(1)与(2)可知

$$
\begin{cases}
\dot V(t)\le-\alpha_1V(t)+\beta_1V(t-\tau), & nT<t\le nT+\delta,\\
\dot V(t)\le\alpha_2V(t)+\beta_2V(t-\tau), & nT+\delta<t\le(n+1)T.
\end{cases}\tag{18}
$$

利用引理3与引理4不难解得, 对于任意的正整数n 而言

$$
\begin{cases}
V(t)\le V_{nT}e^{-\gamma(t-nT)}, & nT<t\le nT+\delta,\\
V(t)\le V_{nT+\delta}e^{\eta(t-nT-\delta+\tau)}, & nT+\delta<t\le(n+1)T,
\end{cases}\tag{19}
$$

其中: γ和η分别是方程$\alpha_1-\gamma-\beta_1e^{\gamma\tau}=0$和$\alpha_2+\beta_2e^{\eta\tau}=\eta$的解.

容易看出, $V(t)$在$(nT\le t\le nT+\delta]$上是非增函数, 在$(nT+\delta\le t\le(n+1)T]$上是非减函数. 并在$t=nT$时, $V_{nT}=\sup\limits_{nT-\tau\le t\le nT}V(t)$, 在$t=nT+\delta$时, $V_{nT+\delta}=\sup\limits_{nT+\delta-\tau\le t\le nT+\delta}V(t)$.

由于时滞τ的尺度大小并无特殊限制, 所以$nT-\tau$有可能属于区间$((n-1)T,(n-1)T+\delta]$, 也有可能属于区间$((n-1)T+\delta,nT]$, 同样$nT+\delta-\tau$有可能属于区间$((n-1)T+\delta,nT]$或$(nT,nT+\delta]$. 因此, 以下证明过程需分$\delta-\tau>0$与$\delta-\tau<0$两种情形:

情形1: 当$\delta-\tau>0$时, 我们须证明

$$
\begin{cases}
V(t) \le V_\tau e^{-[\gamma t-n\gamma(T-\delta+\tau)-n(T-\delta+\tau)]}, & t\in(nT,nT+\delta],\\
V(t) \le V_\tau e^{-\{(n+1)[\gamma(\delta-\tau)-\eta(T-\delta+\tau)]-\eta[t-(n+1)T]\}}, & t\in(nT+\delta,(n+1)T],\\
V_{nT} \le V_\tau e^{-n[\gamma(\delta-\tau)-\eta(T-\delta+\tau)]},\\
V_{nT+\delta} \le V_\tau e^{-[(n+1)\gamma(\delta-\tau)-n\eta(T-\delta+\tau)]}.
\end{cases}\tag{20}
$$

由假设2, 存在$V_\tau \ge V_0$.

下面利用数学归纳法证明.

(i) 当$l=0$时, 即$t\in[0,T]$时, 由(18)式得

$$\begin{cases} V(t)\le V_0 e^{-\gamma t}\le V_\tau e^{-\gamma t}, & 0<t\le\delta,\\ V(t)\le V_\delta e^{\eta(t-\delta+\tau)}, & \delta<t\le T.\end{cases} \tag{21}$$

由(21)式可知, $V_\delta=\sup\limits_{\delta-\tau\le t\le\delta}V(t)\le V(\delta-\tau)\le V_\tau e^{-\gamma(\delta-\tau)}$, 则当$t\in[\delta,T]$时, $V(t)\le V_\tau e^{-\{[\gamma(\delta-\tau)-\eta(T-\delta+\upsilon)]-\eta(t-T)\}}$. 于是有

$$\begin{cases} V(t)\le V_\tau e^{-\gamma t}, & 0<t\le\delta,\\ V(t)\le V_\tau e^{-\{[\gamma(\delta-\tau)-\eta(T-\delta+\tau)]-\eta(t-T)\}}, & \delta<t\le T.\end{cases} \tag{22}$$

(ii) 假设当$l=n-1$时, 下面不等式组成立

$$\begin{cases} V(t) & \le V_\tau e^{-[\gamma t-(n-1)\gamma(T-\delta+\tau)-(n-1)(T-\delta+\tau)]}, \quad t\in((n-1)T,(n-1)T+\delta],\\ V(t) & \le V_\tau e^{-\{n[\gamma(\delta-\tau)-\eta(T-\delta+\tau)]-\eta[t-nT]\}}, \quad t\in((n-1)T+\delta,nT],\\ V_{(n-1)T} & \le V_\tau e^{-(n-1)[\gamma(\delta-\tau)-\eta(T-\delta+\tau)]},\\ V_{(n-1)T+\delta} & \le V_\tau e^{-[n\gamma(\delta-\tau)-(n-1)\eta(T-\delta+\tau)]}.\end{cases} \tag{23}$$

推导当$l=n$时, (23)式成立.

我们知道, 当$l=n$时, $t\in(nT,(n+1)T]$. 由(18)式, 则有

$$\begin{cases} V(t)\le V_{nT}e^{-\gamma t(t-nT)}, & nT\le t\le nT+\delta,\\ V(t)\le V_{nT+\delta}e^{\eta(t-nT+\delta-\tau)}, & nT+\delta<t\le(n+1)T.\end{cases} \tag{24}$$

只要推出V_{nT}与$V_{nT+\delta}$的表达式, 即得(24)式. 由引理4知,

$$\begin{aligned} V_{nT} &= \sup_{nT-\tau\le t\le nT}V(t)\\ &=\max\{\sup_{nT-\tau\le t\le(n-1)T+\delta}V(t),\sup_{(n-1)T+\delta\le t\le nT}V(t)\}\\ &=\max\{V(nT-\tau),V(nT)\}.\end{aligned}$$

因为$(n-1)T<nT-\tau\le(n-1)T+\delta$, 由式(24)可得: $V_{nT}\le V_\tau e^{-n[\gamma(\delta-\tau)-\eta(T-\delta+\tau)]}$. 另外, $V_{nT+\delta}=\sup\limits_{nT+\delta-\tau\le t\le nT+\delta}V(t)\le V(nT+\delta-\tau)$. 由式(23)知: $V_{nT+\delta}\le V_{nT}e^{-\gamma(\delta-\tau)}\le V_\tau e^{-[(n+1)\gamma(\delta-\tau)-n\eta(T-\delta+\tau)]}$.

由上面分析过程, 得到如下结果:

对任意$t\ge0$, 总存在$n_0\in Z^+$, 使得当$t\in[n_0T,n_0T+\delta)$时, 取$\frac{t-\delta}{T}\le n_0\le\frac{t}{T}$, 则在情形1下, 当$\delta-\tau>0$时, 有

$$\begin{aligned} V(t) &\le V_\tau e^{-[\gamma n_0T-n_0\gamma(T-\delta+\tau)-n_0\eta(T-\delta+\tau)]}\\ &\le V_\tau e^{-[\frac{t-\delta}{T}\gamma T-\frac{t}{T}\gamma(T-\delta+\tau)-\frac{t}{T}\eta(T-\delta+\tau)]}\\ &=V_\tau e^{-[\frac{t}{T}(\gamma+\eta)(\delta-\tau)-\eta t-\gamma\delta]},\end{aligned} \tag{25}$$

$$\begin{aligned} V(t) &\le V_\tau e^{-[(n_0+1)\gamma(\delta-\tau)-\eta(T-\delta+\tau)]}\\ &\le V_\tau e^{-[\frac{t}{T}(\gamma(\delta-\tau)-\eta(T-\delta+\tau)]}.\end{aligned} \tag{26}$$

综合上述分析, 总能得出$V(t)\le V_\tau e^{(\frac{t-\delta}{T}+1)\eta(T-\delta+\tau)}$.

因而, 对$t>0$, 意味着

$$\|e^T(t)e(t)+z^T(t)z(t)\|\le\|e^T(t)e(t)\|^2+\|z^T(t)z(t)\|^2\le V_\tau e^{-[\frac{t}{T}(\gamma(\delta-\tau)-\eta(T-\delta+\tau)]}.$$

根据条件(5)(i), 我们得到 $\lim\limits_{t\to+\infty}\|e(t)\|=0$, $\lim\limits_{t\to+\infty}\|z(t)\|=0$, 从而得知误差系统是全局指数稳定的.

情形2: 对于$0<\delta<\tau<T$时, 根据条件(5)(ii), 可以直接由引理5得$V(t)\le V_\tau e^{-\rho t}$. 从而$\lim\limits_{t\to+\infty}\|e(t)\|=0$, $\lim\limits_{t\to+\infty}\|z(t)\|=0$.

注记 2 在定理1中, 我们通过τ和δ之间的大小关系, 分两部分证明了定理的结论. 对于小时滞($\tau<\delta$)的情况, 只需利用引理3和引理4, 即可得到系统达到同步的条件. 对于$\tau\ge\delta$的情况, 由于不能直接由引理3和引理4得到相应的结果, 因此, 我们给出了引理5, 根据引理5, 可以直接获得系统达到同步的相关准则. 在文献[17]中, 作者只考虑了$\tau<\delta$(小时滞)的情况, 给出了一些间歇控制同步准则. 显然, 本文结论比文献[17]更具一般性.

注记 3 本文所选择的控制器$u_i(t)$, $i=1,2$均为线性间歇控制器. 而在实际应用中, 对于非线性间歇控制器也有类似的结论. 例如, 选择

$$u_1(t)=\begin{cases}-Q(e(t)), & t\in(nT,nT+\delta],\\ 0, & t\in(nT+\delta,(n+1)T],\end{cases}$$

$$u_2(t)=\begin{cases}-Q(z(t)), & t\in(nT,nT+\delta],\\ 0, & t\in(nT+\delta,(n+1)T].\end{cases}$$

只需假设连续函数$Q(\cdot)$满足条件$e^T(t)Q(e(t))\le e^T(t)Q_1e(t), z^T(t)Q(z(t))\le z^T(t)Q_2z(t)$.

特别地, 如果模型(3)中的神经元传输函数不存在时滞, 则模型(3)退化为以下形式

$$\begin{cases}STM:\varepsilon\dot{x}(t)=-Ax(t)+Df(x(t))+BS(t),\\ LTM:\dot{S}(t)=-CS(t)+f(x(t)).\end{cases}\tag{27}$$

因此, 从系统描述如下

$$\begin{cases}STM:\varepsilon\dot{y}(t)=-Ay(t)+Df(y(t))+BR(t)+u_1(t),\\ LTM:\dot{R}(t)=-CR(t)+f(y(t))+u_2(t),\end{cases}\tag{28}$$

其中: $u_i(t)$, $i=1,2$ 为间歇控制器

$$u_1(t)=\begin{cases}-Q_1e(t), & nT\le t<nT+\delta,\\ 0, & nT+\delta\le t<(n+1)T,\end{cases}\tag{29}$$

$$u_2(t)=\begin{cases}-Q_2z(t), & nT\le t<nT+\delta,\\ 0, & nT+\delta\le t<(n+1)T,\end{cases}\tag{30}$$

$Q_i\in R^{N\times N}$, $i=1,2$为反馈控制增益矩阵. 令$e(t)=y(t)-x(t)$, $z(t)=R(t)-S(t)$, 则误差动力系统可写为

$$\begin{cases}STM:\varepsilon\dot{e}(t)=-Ae(t)+Dg(e(t))+Bz(t)+u_1(t),\\ LTM:\dot{z}(t)=-Cz(t)+g(e(t))+u_2(t),\end{cases}\tag{31}$$

其中: $g(e(t))=f(y(t))-f(x(t))$.

推论1 若假设1成立. 如果存在正常数α_1, α_2, ε_i, μ_i $(i=1,2,3)$以及正定矩阵Q_1, Q_2满足以下条件

(1)$-2A-2Q_1+\varepsilon_1^{-1}D^TD+\varepsilon_1L^2+\varepsilon\varepsilon_2L^2+\varepsilon_3^{-1}I+\varepsilon\alpha_1I\le0$,

(2)$-2C-2Q_2+\varepsilon^{-1}\varepsilon_2^{-1}I+\varepsilon^{-1}\varepsilon_3B^TB+\alpha_1I\le0$,

(3)$-2A+\mu_1^{-1}D^TD+\mu_1L^2+\varepsilon\mu_2L^2+\mu_3^{-1}I-\varepsilon\alpha_2I\le0$,

(4)$-2C+\varepsilon^{-1}\mu_2^{-1}I+\varepsilon^{-1}\mu_3B^TB-\alpha_2I\le0$,

(5)$\alpha_1\delta-\alpha_2(T-\delta)>0$,

则系统(27)和(28)在间歇控制协议下实现同步.

证明 构造Lyapunov函数$V(t)=e^T(t)e(t)+z^T(t)z(t)$, 当$t\in[nT,nT+\delta)$时, 关于$V(t)$求导数得

$$
\begin{aligned}
\dot V(t)&=2e^T(t)\dot e(t)+2z^T(t)\dot z(t)\\
&=\frac{2}{\varepsilon}e^T(t)(-Ae(t)+Dg(e(t))+Bz(t)-Q_1e(t))\\
&\quad+2z^T(t)(-Cz(t)+g(e(t)))-Q_2z(t))\\
&\le\frac{1}{\varepsilon}e^T(t)(-2A-2Q_1+\varepsilon_1^{-1}D^TD+\varepsilon_1L^2+\varepsilon\varepsilon_2L^2+\varepsilon_3^{-1}I)e(t)\\
&\quad+z^T(t)(-2C-2Q_2+\varepsilon^{-1}\varepsilon_2^{-1}I+\varepsilon_3B^TB)z(t)+\varepsilon^{-1}\varepsilon_2e^T(t-\tau)L^2e(t-\tau)\\
&\le-\alpha_1V(t)+\frac{1}{\varepsilon}e^T(t)(-2A-2Q_1+\varepsilon_1^{-1}D^TD+\varepsilon_1L^2+\varepsilon\varepsilon_2L^2+\varepsilon_3^{-1}I\\
&\quad+\varepsilon\alpha_1I)e(t)+z^T(t)(-2C-2Q_2+\varepsilon^{-1}\varepsilon_2^{-1}I+\varepsilon^{-1}\varepsilon_3B^TB+\alpha_1I)z(t)\\
&\le-\alpha_1V(t).
\end{aligned}
$$

因此$V(t)\le V(nT)e^{-\alpha_1(t-nT)}$, 当$t\in[nT+\delta,(n+1)T)$时, 亦有

$$
\begin{aligned}
\dot V(t)&=2e^T(t)\dot e(t)+2z^T(t)\dot z(t)\\
&=2e^T(t)(-Ae(t)+Dg(e(t))+Bz(t))+2z^T(t)(-Cz(t)+g(e(t))))\\
&\le\frac{1}{\varepsilon}e^T(t)(-2A+\mu_1^{-1}D^TD+\mu_1L^2++\varepsilon\mu_2L^2+\mu_3^{-1}I)e(t)\\
&\quad+z^T(t)(-2C+\varepsilon^{-1}\mu_2^{-1}I+\varepsilon^{-1}\mu_3B^TB)z(t)\\
&\le\alpha_2V(t)+e^T(t)(-2A+\mu_1^{-1}D^TD+\mu_1L^2+\varepsilon\mu_2L^2+\mu_3^{-1}I\\
&\quad-\varepsilon\alpha_2I)e(t)+z^T(t)(-2C+\varepsilon^{-1}\mu_2^{-1}I+\varepsilon^{-1}\mu_3B^TB-\alpha_2I)z(t)\\
&\le\alpha_2V(t).
\end{aligned}
$$

则

$$
\begin{aligned}
V(t)&\le V(nT+\delta)e^{\alpha_2(t-(nT+\delta))}\\
&\le V(nT)e^{-\alpha_1\delta+\alpha_2(t-(nT+\delta))}.
\end{aligned}
$$

故有$V((n+1)T)\le V(nT)e^{-\alpha_1\delta+\alpha_2(T-\delta)}$.

对$\forall t\in[nT,nT+\delta)$, 利用数学归纳法容易得出

$$
\begin{aligned}
V(t)&\le V(t_0)e^{-n[\alpha_1\delta-\alpha_2(T-\delta)}e^{-\alpha_1(t-nT)}\\
&\le V(t_0)e^{-(\alpha_1\delta-\alpha_2(T-\delta))\frac{t-\delta}{T}}.
\end{aligned}
$$

对$\forall t\in[nT+\delta,(n+1)T)$, 亦有

$$
\begin{aligned}
V(t)&\le V(t_0)e^{-(\alpha_1\delta-\alpha_2(T-\delta))\frac{t-\delta}{T}}\cdot e^{-(\alpha_1\delta-\alpha_2(T-\delta))}\\
&=V(t_0)e^{-(\alpha_1\delta-\alpha_2(T-\delta))(\frac{t-\delta}{T}+1)}.
\end{aligned}
$$

由推论1的条件(5)可知$\lim\limits_{t\to+\infty}V(t)=0$, 从而$\lim\limits_{t\to+\infty}\|e(t)\|=0$, $\lim\limits_{t\to+\infty}\|z(t)\|=0$.

注记 4 在推论1中, 我们要求$0<\delta<T$. 而当$\delta=T$时, 我们发现控制器(5)和(6)将退化为一般的连续控制, 即

$$u_1(t)=-Q_1e(t),\quad u_2(t)=-Q_2z(t).\tag{32}$$

推论 2 若假设1、2成立, 如果存在正常数$\alpha_1,\varepsilon_i(i=1,\cdots,4)$以及正定矩阵$Q_1,Q_2$满足以下条件:

(1)$-2A-2Q_1+\varepsilon_1^{-1}D^TD+\varepsilon_1L^2+\varepsilon_2^{-1}D_\tau^TD_\tau+\varepsilon\varepsilon_3L^2+\varepsilon_4^{-1}I+\varepsilon\alpha_1I\le0$,

(2)$-2C-2Q_2+\varepsilon^{-1}\varepsilon_3^{-1}I+\varepsilon^{-1}\varepsilon_4B^TB+\alpha_1I\le0$,

则系统(3)和系统(4)在控制器(32)下实现同步.

3　数值实例与仿真

我们设计了一类数值实例，通过仿真实例验证了模型的实效性．考虑一类2−D变时滞竞争神经网络的数值算例

$$\begin{cases} STM: \varepsilon\dot{x}(t) = -Ax(t) + Df(x(t)) + D_\tau f((t-\tau)) + BS(t), \\ LTM: \dot{S}(t) = -CS(t) + f(x(t)), \end{cases} \tag{33}$$

其中：$x(t)=(x_1(t),x_2(t))^T$，$S(t)=(s_1(t),s_2(t))^T$，$f(x)=(0.6\tanh(x_1)+0.4\sin(x_1),\ 0.6\tanh(x_2)+0.4\sin(x_2))^T$，$\tau=0.4$，$C=0.8I$，$B=0.3I$，$\varepsilon=0.4$，

$$A=\begin{pmatrix} 1.5 & 0 \\ 0 & 2.2 \end{pmatrix},\quad D=\begin{pmatrix} 1 & -1.8 \\ 1 & 2.5 \end{pmatrix},\quad D_\tau=\begin{pmatrix} -1.45 & -0.05 \\ -0.28 & 0.05 \end{pmatrix}.$$

此动力行为系统的初始条件为$x(0)=(2,0.4)^T$，$S(0)=(-0.5,0.8)^T$，运行结果如图1，图2所示．

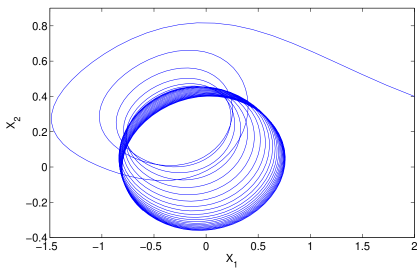

图 1　$x(t)$的相图

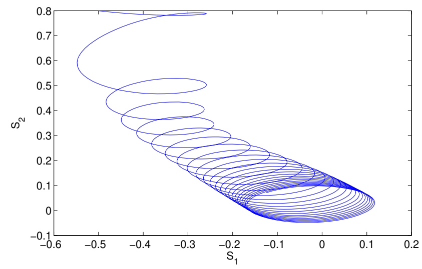

图 2　$S(t)$的相图

考虑主系统(33)与从系统(34)的同步性

$$\begin{cases} STM: \varepsilon\dot{y}(t) = -Ay(t) + Df(y(t)) + D_\tau f((t-\tau)) + BR(t) + u_1(t), \\ LTM: \dot{R}(t) = -CR(t) + f(y(t)) + u_2(t), \end{cases} \tag{34}$$

其中：$y(t)=(y_1(t),y_2(t))^T$，$R(t)=(r_1(t),r_2(t))^T$．初始条件为$y(0)=(-1.6,0.6)^T$，$R(0)=(0.2,0.56)^T$，$x_i(t)$，$y_i(t)$，$s_i(t)$和$r_i(t)(i=1,2)$的状态如图3∼图6呈现了无控制的情况．显然，未控制下主从系统的轨线是不同步的．

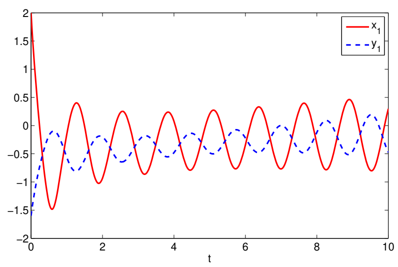

图 3　$x_1(t)$ 和$y_1(t)$ 的状态

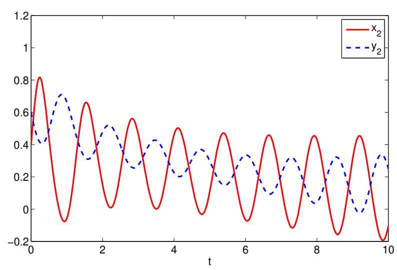

图 4　$x_2(t)$ 和$y_2(t)$ 的状态

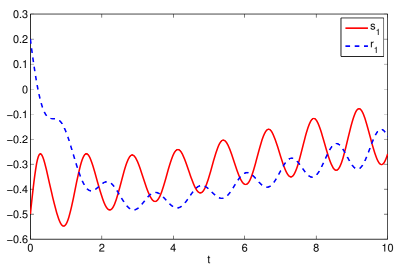

图 5　$s_1(t)$和$r_1(t)$的状态

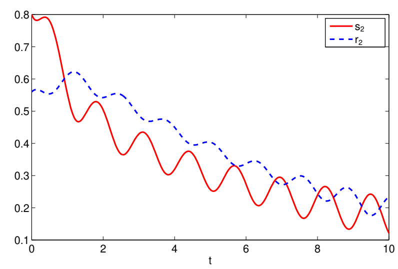

图 6　$s_2(t)$和$r_2(t)$的状态

通过对定理1的分析可知, 在选取适当条件时, 系统在周期间歇控制(5)和(6)下, 主系统(33)与从系统(34)将趋于同步. 为简便起见, 我们在本例中选择了$Q_1=3.5I$, $Q_2=3I$, $\varepsilon_1=\mu_1=2$, $\varepsilon_2=\mu_2=1$, $\varepsilon_3=\mu_3=0.5$, $\varepsilon_4=\mu_4=0.25$. 简单计算后可知定理1的条件(1)~(4)成立. 此外，选择$\delta=2$, $T=3$, 定理1的条件(5)(i)也成立. 故本例选择$\delta=2$, $T=3$, 在周期间歇控制(5)和(6)下, $x_i(t)$, $y_i(t)$, $s_i(t)$与$r_i(t)$的形态轨线如图7~图10所示, 其中$i=1,2$, 此时主从系统趋于同步十分明显.

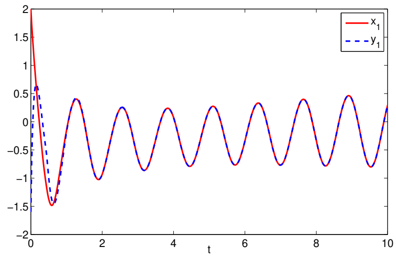

图 7　$x_1(t)$和$y_1(t)$的状态

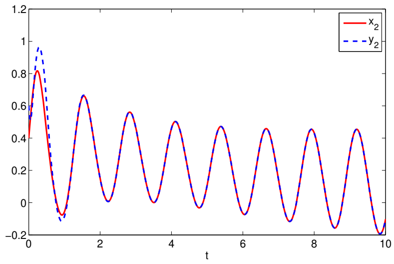

图 8　$x_2(t)$和$y_2(t)$的状态

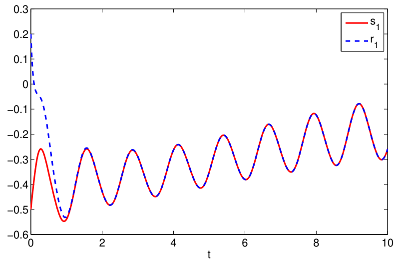

图 9　$s_1(t)$和$r_1(t)$的状态

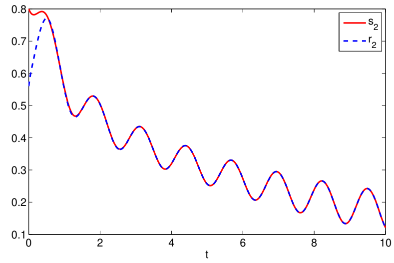

图 10　$s_2(t)$和$r_2(t)$的状态

4　结 论

本文在非零持续时间的间歇反馈控制可以稳定到连续时间的时滞神经网络的基础上, 考虑了具有时变时滞的竞争神经网络模型. 利用微分不等式分析方法和Lyapunov函数法, 讨论了具有时滞的竞争神经网络的全局指数同步条件, 并建立了相应的判别准则. 同时, 也研究了既满足大时滞又满足小时滞的时变时滞和间歇控制的竞争神经网络的指数同步问题, 建立了一些充分性条件, 并通过仿真实例验证了所提出方案的正确性和有效性.

参考文献:

[1] COHEN M, GROSSBERG S. Absolute stability of global pattern formation and parallel memory storage by competitive neural networks[J]. IEEE Trans Syst Man Cybern, 1983, 13: 815–826.

[2] MEI X H, ZHANG L W, YU Z Y, et al. Synchronization of competitive neural networks with time-varying delays and impulses effects[J]. Journal of Xinjiang University(Natural Science Edition), 2016, 33(3): 280-286.

[3] MEYER-BÄSE A, PILYUGIN S, WISMÜLLER A, et al. Local exponential stability of competitive neural networks with different time scales[J]. Eng Appl Artif Intell, 2004, 17: 227–232.

[4] LOU X, CUI B. Synchronization of competitive neural networks with different time scales[J]. Physica A, 2007, 380: 563–576.

[5] MEYER-BÄSE A, ROBERTS R, THÜMMLER V. Local uniform stability of competitive neural networks wiyh different time scales under vanishing perturbations[J]. Neurocomputing, 2010, 73: 770–775.

[6] NIE X B, CAO J D. Exponential stability of competitive neural networks with time-varying and distributed delays[J]. Proceedings Inst Mech E Part I Journal of Systems and Control Engineering, 2008, 222: 583-594.

[7] NIE X B, CAO J D. Multistability of competitive neural networks with time-varying and distributed delays[J]. Nonlinear Analysis: Real World Applications, 2009, 10: 928–942.

[8] MEYER-BÄSE A, THÜMMLER V. Local and global stability of an unsupervised competitive neural networks[J]. IEEE Transactions on Neural Networks, 2008, 19: 346–351.

[9] GU H B. Adaptive synchronization for competitve neural networks with different time scales and stochastic perturbation[J]. Neurocompyting, 2009, 73: 350–356.

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[7] WANG M, LIU Y, HUANG Z. Large margin object tracking with circulant feature maps[C]. Honolulu: 2017 IEEE Conference on Computer Vision and Pattern Recognition, 2017, 4800-4808.

[8] WU Y, LIM J, YANG M H. Object tracking benchmark[J]. IEEE Transactions on Pattern Analysis and Machine Intelligence, 2015, 37(9): 1834-1848.

[9] MUELLER M, SMITH N, GHANEM B. A benchmark and simulator for uav tracking[J]. Far East Journal of Mathematical Sciences, 2016, 2(2): 445-461.

[10] DANELLJAN M, BHAT G, KHAN F S, et al. ECO: efficient convolution operators for tracking[C]. Honolulu: 2017 IEEE Conference on Computer Vision and Pattern Recognition (CVPR), 2017, 6931-6939.

[11] DANELLJAN M, HÄGER G, KHAN F S, et al. Discriminative scale space tracking[J]. IEEE Transactions on Pattern Analysis & Machine Intelligence, 2017, 39(8): 1561-1575.

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[10] NIE X B, CAO J D. Existence and global stability of equilibrium point for delayed competitive neural networks with discontinuous activation function[J]. Int J Syst Sci, 2012, 43: 459–474.

[11] GAN Q T. Synchronization of competitive neural networks with different time scales and time-varying delay based on delay partitioning approach[J]. Int J Mach Learn & Cyber, 2013, 4: 327–337.

[12] PECORA L, CARROLL T. Synchronization in chaotic systems[J]. Phys Rev Lett, 1990, 64: 821–824.

[13] ZHANG H, GUAN Z H, HO D. On synchronization of hybrid switching and impulsive networks[J]. In:45th IEEE conference on decision and control, 2006, 1: 2765–2770.

[14] WEN G H, DUAN Z S, CHEN G R. Distributed consensus of multi-agent systems with general linear node dynamics and intermittent communications[J]. International Journal of Robust Nonlinear Control, 2014, 24(16): 2438–2457.

[15] HUANG N, DUAN Z S, ZHAO Y. Consensus of multi-agent systems via delayed and intermittent communications[J]. IET Control Theory and Applications, 2015, 9(1): 62–73.

[16] LI H J, SU H Y. Distributed consensus of multi-agent systems with nonlinear dynamics via adaptive intermittent control[J]. Journal of the Franklin Institute, 2015, 352(10): 4546–4564.

[17] YU J, HU C, JIANG H J, et al. Synchronization of nonlinear systems with delays via periodically nonliear intermittent control[J]. Commun Nonlinear SCI Number Simulat, 2012, 17: 2978–2989.

[18] LI C D, FENG G, LIAO X F. Stabilization of nonlinear systems via periodically intermittent control[J]. IEEE Trans Circ Syst-II, 2007, 54: 1019–1023.

[19] HUANG J J, LI C D, HAN Q. Stabilization of delayed chaotic neural networks by periodically intermittent control[J]. Circ Syst Signal Pr, 2009, 28: 567–579.

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