

# Consensus of Multi-Agent Systems with Directed Intermittent Communications\*

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**Abstract:** This paper studies the consensus of second-order multi-agent systems (MASs) with general directed intermittent communications. Firstly, a directed spanning tree transformation method is applied to get the consensus error system. Moreover, some consensus conditions based on the coupling strengths and the communication width are obtained by using the Lyapunov stability theory and matrix inequalities. Finally, a numerical example is given to verify the theoretical result.

**Key words:** multi-agent systems; consensus; directed intermittent communications

**DOI:** 10.13568/j.cnki.651094.651316.2020.04.26.0002

**CLC number:** O231.5 **Document Code:** A **Article ID:** 2096-7675(2021)03-0257-012

**Citation format:** YU Z Y, YU S Z, JIANG H J. Consensus of multi-agent systems with directed intermittent communications[J]. Journal of Xinjiang University(Natural Science Edition in Chinese and English), 2021, 38(3): 257-268+284.

## 0 Introduction

Last decades have witnessed the rapid development of cooperative control for MASs. This research field includes consensus control, rendezvous control, formation control and so on. The consensus of MASs, as one of the most important research topics, has received much attention since it has been widely applied to various areas, such as traffic control, sensor networks, mobile robots and unmanned aerial vehicles<sup>[1-4]</sup>.

The consensus means that a number of agents achieve agreement about some variables of interests, such as position, velocity, and attitude by communicating with all agents. The main task of consensus is to devise the protocols which only rely on neighbor's information. One pioneer work on consensus of MASs is [5], in which a general framework of first-order consensus was proposed. The existing consensus works can be simply categorised into leaderless consensus and leader-follower consensus. In [6], the intermittent saturated controller was designed to study the semi-global leader-following consensus. The leader-following consensus with intermittent control and adaptive control was considered in [7]. These works concerned leader-following consensus on directed graphs. However, many research works on leaderless consensus of MASs mainly focused on some special graphs, such as undirected graph<sup>[8]</sup>, strongly connected graph<sup>[9]</sup>, balanced graph<sup>[10]</sup>, and so on. For general digraphs, the linear MASs with periodic intermittent communication was considered in [11]. However, there is still little work on consensus of second-order MASs with aperiodically intermittent communication and general nonlinear dynamics.

This paper aims at proposing a novel method to study consensus of second-order MASs with general directed communication. Compared with existing works, the main contributions of this paper can be summarized as follows: (1) According to the reordering scheme, a directed spanning tree transformation method is derived, which can transform the consensus problem into the stability of the zero solution of the error system. (2) For second-order MASs with general digraphs and nonlinear dynamics, a directed and aperiodically intermittent communication protocol is designed and some criteria for reaching consensus are obtained.

\* **Received Date:** 2020-04-26

**Foundation Item:** This work was supported by the Tianshan Youth Program(2018Q068), the Tianshan Xuesong Program (2018XS02) and National Natural Science Foundation of China(62003289; U1703262).

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# 1 Preliminaries and Model formulation

For convenience, some notations are given in this section.  $R^n$  is the  $n$ -dimensional Euclidean space.  $\mathbb{N}_+$  denotes the set of positive integer numbers.  $\mathbf{I}_N$  is the identity matrix with dimension  $N$ .  $\mathbf{1}$  and  $\mathbf{0}$  are column vectors with each entry being 1 and 0, respectively.  $\mathbf{O}_N$  denotes the  $N$ -dimensional zero matrix. For a matrix  $\mathbf{C}$ ,  $C_{ij}$  represents its entry in the  $i$ th row and  $j$ th column. For a matrix  $\mathbf{B}$ ,  $\mathbf{B}^T$ ,  $\lambda_{\min}(\mathbf{B})$  and  $\lambda_{\max}(\mathbf{B})$  represent its transpose, the minimum and the maximum eigenvalues of  $\mathbf{B}$ , respectively.  $\mathbf{B} > 0$  means  $\mathbf{B}$  is a positive definite matrix.  $\|\cdot\|$  is the Euclidean norm.  $\otimes$  represents the Kronecker product.  $\text{diag}(\cdot)$  denotes the diagonal matrix.

Consider a second-order MAS with inherent nonlinear dynamics, in which the  $i$ -th agent is described by

$$\begin{aligned} \dot{p}_i(t) &= q_i(t), \\ \dot{q}_i(t) &= f(t, p_i(t), q_i(t)) + u_i(t), \quad i = 1, 2, \dots, N \end{aligned} \quad (1)$$

where  $p_i(t) \in R^n$ ,  $q_i(t) \in R^n$ , and  $u_i(t) \in R^n$  are the position state, velocity state, and the control input.  $f: R^+ \times R^n \times R^n \rightarrow R^n$  is a continuously differentiable vector-valued function which represents the inherent nonlinear dynamics.

In the following, some related definitions and assumptions are given.

**Definition 1**<sup>[12]</sup> Let  $\mathbf{M}_1^N(1) = \{\tilde{\mathbf{M}} : \tilde{\mathbf{M}} \in R^{(N-1) \times N}\}$ , in which the  $i$ -th row of  $\tilde{\mathbf{M}}$  has only one entry  $\alpha_i$  and only one entry  $-\alpha_i$  with  $\alpha_i \neq 0$ , and other entries are zeros. Let  $\mathbf{M}_1^N(n) = \{\tilde{\mathbf{M}} \otimes I_n : \tilde{\mathbf{M}} \in \mathbf{M}_1^N(1)\}$ .

**Definition 2**<sup>[13]</sup> The consensus for second-order MAS (1) is said to be reached if for any initial conditions,

$$\lim_{t \rightarrow \infty} \|p_i(t) - p_j(t)\| = 0, \quad \lim_{t \rightarrow \infty} \|q_i(t) - q_j(t)\| = 0,$$

where  $i, j = 1, 2, \dots, N$ .

**Assumption 1** For any  $p, y, q, \mu \in R^n$  and  $t \geq 0$ , there exist two nonnegative constants  $\rho_1$  and  $\rho_2$  such that

$$\|f(t, p, q) - f(t, y, \mu)\| \leq \rho_1 \|p - y\| + \rho_2 \|q - \mu\|.$$

**Assumption 2** The communication graph  $\mathbf{G} = (\mathbf{V}, \mathbf{E}, \mathbf{A})$  contains a directed spanning tree, and  $\bar{\mathbf{G}} = (\mathbf{V}, \bar{\mathbf{E}}, \bar{\mathbf{A}})$  is a directed spanning tree of  $\mathbf{G}$ .

**Remark 1** The reordering scheme proposed in [12] is also applied. In the directed spanning tree  $\bar{\mathbf{G}}$ , all nodes are divided into three categories: the root, the stem, and the leaf. In the directed spanning tree  $\bar{\mathbf{G}}$ , we denote  $v_{ik}$  as the parent node of node  $v_{k+1}$  for  $1 \leq k \leq N-1$ , then  $(v_{ik}, v_{k+1})$  is a directed edge of  $\bar{\mathbf{G}}$ . Specially, when  $k = 1$ ,  $v_{ik} = v_1$ , and  $v_{i_{k-1}}$  represents the parent node of  $v_{ik}$ . Then the following lemmas are given.

**Lemma 1**<sup>[12]</sup> Suppose that the digraph  $\mathbf{G} = (\mathbf{V}, \mathbf{E}, \mathbf{A})$  contains a directed spanning tree  $\bar{\mathbf{G}} = (\mathbf{V}, \bar{\mathbf{E}}, \bar{\mathbf{A}})$ , where  $\bar{\mathbf{E}} = \{(v_{ik}, v_{k+1}) | k = 1, 2, \dots, N-1\}$ ,  $\bar{\mathbf{A}} = [\bar{a}_{ij}]_{N \times N}$  is the weighted adjacent matrix. The element of  $\bar{\mathbf{A}}$  satisfies  $\bar{a}_{ij} > 0$  if  $(v_j, v_i) \in \bar{\mathbf{E}}$  and  $\bar{a}_{ij} = 0$ , otherwise. Let  $\mathbf{H} \in \mathbf{M}_1^N(1)$  be an  $(N-1) \times N$  matrix and its element satisfies

$$\mathbf{H}_{ij} = \begin{cases} -1, & j = i + 1, \quad i = 1, 2, \dots, N-1, \\ 1, & \bar{a}_{i+1, j} > 0, \quad i, j = 1, 2, \dots, N-1, \\ 0, & \text{otherwise.} \end{cases}$$

Then, the consensus for second-order MAS (1) can be realized if and only if

$$\lim_{t \rightarrow \infty} \|(\mathbf{H} \otimes \mathbf{I}_n) p(t)\| = 0, \quad \lim_{t \rightarrow \infty} \|(\mathbf{H} \otimes \mathbf{I}_n) q(t)\| = 0, \quad (2)$$

where  $p(t) = (p_1^T(t), p_2^T(t), \dots, p_N^T(t))^T$  and  $q(t) = (q_1^T(t), q_2^T(t), \dots, q_N^T(t))^T$ .

**Lemma 2**<sup>[12]</sup> Suppose that Assumption 2 is satisfied and  $\mathbf{L} \in R^{N \times N}$  is the Laplacian matrix of  $\mathbf{G}$ . Constructing a matrix  $\mathbf{K} \in R^{(N-1) \times (N-1)}$  which satisfies  $\mathbf{HL} = \mathbf{KH}$ . Then, the matrix  $\mathbf{K} = \mathbf{HLS}$  has the same eigenvalues with the Laplacian matrix  $\mathbf{L}$

except for zero eigenvalue. In addition, the element of  $\mathbf{S}$  satisfies

$$\mathbf{S}_{ij} = \begin{cases} \sum_{l=1}^{i-1} \mathbf{H}_{i-1,l} \mathbf{S}_{lj}, & 1 \leq j < i-1, \quad i=2, \dots, N, \\ 0, & j=i-1, \quad i=2, \dots, N, \\ 1, & j \geq i, \quad i=1, 2, \dots, N-1. \end{cases}$$

**Lemma 3** If Assumption 1 holds, then for any matrix  $\mathbf{\Lambda} > 0$ , there exists a symmetric matrix  $\mathbf{P} > 0$  such that

$$\mathbf{P}(-\mathbf{K}) + (-\mathbf{K})^T \mathbf{P} = -\mathbf{\Lambda}. \quad (3)$$

**Proof** Because of  $\mathbf{G}$  containing a directed spanning tree, all eigenvalues of  $\mathbf{L}$  have positive real parts except for a simple zero eigenvalue<sup>[14]</sup>. Based on the analysis of Lemma 2, all eigenvalues of  $\mathbf{K}$  have positive real parts, so the matrix  $-\mathbf{K}$  is Hurwitz. Hence, there exists a symmetric positive definite solution  $\mathbf{P}$  satisfying the Lyapunov equation (3).

## 2 Main Results

The system (1) with aperiodically intermittent communication is described as follows

$$\begin{cases} \dot{p}_i(t) = q_i(t) \\ \dot{q}_i(t) = f(t, p_i(t), q_i(t)) - \alpha \sum_{j=1}^N \mathbf{L}_{ij} p_j(t) - \beta \sum_{j=1}^N \mathbf{L}_{ij} q_j(t), \quad t \in [t_m, t_m + \delta_m), \\ \dot{q}_i(t) = f(t, p_i(t), q_i(t)), \quad t \in [t_m + \delta_m, t_{m+1}), \end{cases} \quad (4)$$

where  $\alpha$  and  $\beta$  are coupling strengths,  $\delta_m$  denotes the communication width in the  $m$ th time interval,  $m \in \mathbb{N}_+$  and  $t_1 = 0$ .

Let  $\bar{p}_k(t) = p_{i_k}(t) - p_{k+1}(t)$  and  $\bar{q}_k(t) = q_{i_k}(t) - q_{k+1}(t)$  be the error states of the position and the velocity for  $1 \leq k \leq N-1$ . Then,

$$\bar{p}(t) = (\mathbf{H} \otimes \mathbf{I}_n) p(t), \quad \bar{q}(t) = (\mathbf{H} \otimes \mathbf{I}_n) q(t) \quad (5)$$

where  $\bar{p}(t) = (\bar{p}_1^T(t), \bar{p}_2^T(t), \dots, \bar{p}_{N-1}^T(t))^T$ ,  $p(t) = (p_1^T(t), p_2^T(t), \dots, p_N^T(t))^T$ ,  $\bar{q}(t) = (\bar{q}_1^T(t), \bar{q}_2^T(t), \dots, \bar{q}_{N-1}^T(t))^T$ ,  $q(t) = (q_1^T(t), q_2^T(t), \dots, q_N^T(t))^T$ . According to Lemma 1, one can get that the consensus is reached if and only if

$$\lim_{t \rightarrow \infty} \|\bar{p}(t)\| = 0, \quad \lim_{t \rightarrow \infty} \|\bar{q}(t)\| = 0. \quad (6)$$

The error dynamics of system (4) is written as

$$\begin{cases} \dot{\bar{p}}_k(t) = \bar{q}_k(t) \\ \dot{\bar{q}}_k(t) = \bar{f}(t, \bar{p}_k(t), \bar{q}_k(t)) - \alpha \sum_{j=1}^N (\mathbf{L}_{i_k,j} - \mathbf{L}_{k+1,j}) p_j(t) - \beta \sum_{j=1}^N (\mathbf{L}_{i_k,j} - \mathbf{L}_{k+1,j}) q_j(t), \quad t \in [t_m, t_m + \delta_m), \\ \dot{\bar{q}}_k(t) = \bar{f}(t, \bar{p}_k(t), \bar{q}_k(t)), \quad t \in [t_m + \delta_m, t_{m+1}), \end{cases} \quad (7)$$

where  $\bar{f}(t, \bar{p}_k(t), \bar{q}_k(t)) = f(t, p_{i_k}(t), q_{i_k}(t)) - f(t, p_{k+1}(t), q_{k+1}(t))$ .

Let  $F(t, \bar{p}(t), \bar{q}(t)) = (\bar{f}^T(t, \bar{p}_1(t), \bar{q}_1(t)), \dots, \bar{f}^T(t, \bar{p}_{N-1}(t), \bar{q}_{N-1}(t)))^T$ . The compact form of the error dynamics can be written as

$$\begin{cases} \dot{\bar{p}}(t) = \bar{q}(t) \\ \dot{\bar{q}}(t) = F(t, \bar{p}(t), \bar{q}(t)) - \alpha (\mathbf{H}\mathbf{L} \otimes \mathbf{I}_n) p(t) - \beta (\mathbf{H}\mathbf{L} \otimes \mathbf{I}_n) q(t), \quad t \in [t_m, t_m + \delta_m), \\ \dot{\bar{q}}(t) = F(t, \bar{p}(t), \bar{q}(t)), \quad t \in [t_m + \delta_m, t_{m+1}). \end{cases} \quad (8)$$

**Theorem 1** Under Assumption 1 and Assumption 2, the second-order MAS (4) can achieve consensus if the following conditions are satisfied

$$\lambda_{\min}(\mathbf{\Lambda}) > \max \left\{ \frac{\xi_1}{\alpha^2}, \frac{\xi_2}{\beta^2} \right\}, \quad (9)$$

$$\delta_m > \frac{\eta_2}{\eta_1 + \eta_2} (t_{m+1} - t_m), \quad (10)$$

where  $\Lambda = \mathbf{PK} + \mathbf{K}^T \mathbf{P}$ ,  $\mathbf{P} > 0$ ,  $\eta_1 = \lambda_{\min}(\Theta^{-1} \mathbf{R}_1)$ ,  $\eta_2 = \lambda_{\max}(\Theta^{-1} \mathbf{R}_2)$ ,  $\xi_1 = [\alpha(2\rho_1 + \rho_2) + \beta\rho_1] \lambda_{\max}(\mathbf{P})$ ,  $\xi_2 = [\beta(2\rho_2 + \rho_1) + \alpha(\rho_2 + 2)] \lambda_{\max}(\mathbf{P})$ ,  
 $\mathbf{R}_1 = \begin{pmatrix} \alpha^2 \Lambda - \xi_1 \mathbf{I}_{N-1} & \mathbf{O}_{N-1} \\ \mathbf{O}_{N-1} & \beta^2 \Lambda - \xi_2 \mathbf{I}_{N-1} \end{pmatrix}$ ,  $\Theta = \begin{pmatrix} \alpha\beta\Lambda & \alpha\mathbf{P} \\ \alpha\mathbf{P} & \beta\mathbf{P} \end{pmatrix}$ ,  $\mathbf{R}_2 = \begin{pmatrix} \xi_1 \mathbf{I}_{N-1} & \alpha\beta\Lambda \\ \alpha\beta\Lambda & \xi_2 \mathbf{I}_{N-1} \end{pmatrix}$ .

**Proof** We consider the following Lyapunov function candidate

$$V(t) = \frac{1}{2} (\bar{p}^T(t), \bar{q}^T(t)) (\Theta \otimes \mathbf{I}_n) \begin{pmatrix} \bar{p}(t) \\ \bar{q}(t) \end{pmatrix}, \quad (11)$$

where  $\Theta = \begin{pmatrix} \alpha\beta\Lambda & \alpha\mathbf{P} \\ \alpha\mathbf{P} & \beta\mathbf{P} \end{pmatrix}$ ,  $\Lambda = \mathbf{PK} + \mathbf{K}^T \mathbf{P}$ ,  $\mathbf{P} > 0$ ,  $\alpha > 0$ , and  $\beta > 0$ . Based on inequality (9), one can easily obtain that  $\Theta > 0$ . Then,  $V(t) \geq 0$ , and  $V(t) = 0$  if and only if  $\bar{p}(t) = \mathbf{0}_{n(N-1)}$ ,  $\bar{q}(t) = \mathbf{0}_{n(N-1)}$ . For  $t \in [t_m, t_m + \delta_m)$  and  $m \in \mathbb{N}_+$ , taking the derivative of  $V(t)$  along the trajectories of (8) and combination with Lemma 2 and Lemma 3, we obtain

$$\begin{aligned} \dot{V}(t) &= \alpha\beta\bar{p}^T(t)(\Lambda \otimes \mathbf{I}_n)\bar{q}(t) + \alpha\bar{q}^T(t)(\mathbf{P} \otimes \mathbf{I}_n)\bar{q}(t) + [\alpha\bar{p}^T(t) + \beta\bar{q}^T(t)] \\ &\quad \times (\mathbf{P} \otimes \mathbf{I}_n)F(t, \bar{p}(t), \bar{q}(t)) - \alpha^2\bar{p}^T(t)(\mathbf{PHL} \otimes \mathbf{I}_n)\bar{p}(t) - \alpha\beta\bar{p}^T(t)(\mathbf{PHL} \otimes \mathbf{I}_n)\bar{q}(t) \\ &\quad - \alpha\beta\bar{q}^T(t)(\mathbf{PHL} \otimes \mathbf{I}_n)\bar{p}(t) - \beta^2\bar{q}^T(t)(\mathbf{PHL} \otimes \mathbf{I}_n)\bar{q}(t) \\ &= \alpha\beta\bar{p}^T(t)(\Lambda \otimes \mathbf{I}_n)\bar{q}(t) + \alpha\bar{q}^T(t)(\mathbf{P} \otimes \mathbf{I}_n)\bar{q}(t) + [\alpha\bar{p}^T(t) + \beta\bar{q}^T(t)] \\ &\quad \times (\mathbf{P} \otimes \mathbf{I}_n)F(t, \bar{p}(t), \bar{q}(t)) - \frac{\alpha^2}{2}\bar{p}^T(t)(\Lambda \otimes \mathbf{I}_n)\bar{p}(t) - \alpha\beta\bar{p}^T(t)(\mathbf{PK} \otimes \mathbf{I}_n)\bar{q}(t) \\ &\quad - \alpha\beta\bar{q}^T(t)(\mathbf{PK} \otimes \mathbf{I}_n)\bar{p}(t) - \frac{\beta^2}{2}\bar{q}^T(t)(\Lambda \otimes \mathbf{I}_n)\bar{q}(t) \\ &= [\alpha\bar{p}^T(t) + \beta\bar{q}^T(t)](\mathbf{P} \otimes \mathbf{I}_n)F(t, \bar{p}(t), \bar{q}(t)) - \frac{\alpha^2}{2}\bar{p}^T(t)(\Lambda \otimes \mathbf{I}_n)\bar{p}(t) \\ &\quad - \frac{\beta^2}{2}\bar{q}^T(t)(\Lambda \otimes \mathbf{I}_n)\bar{q}(t) + \alpha\bar{q}^T(t)(\mathbf{P} \otimes \mathbf{I}_n)\bar{q}(t). \end{aligned}$$

Based on Assumption 1, it has

$$\begin{aligned} &[\alpha\bar{p}^T(t) + \beta\bar{q}^T(t)](\mathbf{P} \otimes \mathbf{I}_n)F(t, \bar{p}(t), \bar{q}(t)) \\ &\leq \lambda_{\max}(\mathbf{P})(\alpha\|\bar{p}(t)\| + \beta\|\bar{q}(t)\|)\|F(t, \bar{p}(t), \bar{q}(t))\| \\ &\leq \lambda_{\max}(\mathbf{P})(\alpha\|\bar{p}(t)\| + \beta\|\bar{q}(t)\|)(\rho_1\|\bar{p}(t)\| + \rho_2\|\bar{q}(t)\|) \\ &\leq \left(\alpha\rho_1 + \frac{\alpha\rho_2 + \beta\rho_1}{2}\right) \lambda_{\max}(\mathbf{P})\|\bar{p}(t)\|^2 + \left(\beta\rho_2 + \frac{\alpha\rho_2 + \beta\rho_1}{2}\right) \lambda_{\max}(\mathbf{P})\|\bar{q}(t)\|^2. \end{aligned} \quad (12)$$

In combination with (13), the equation (12) is written as

$$\begin{aligned} \dot{V}(t) &\leq \alpha\bar{q}^T(t)(\mathbf{P} \otimes \mathbf{I}_n)\bar{q}(t) - \frac{\alpha^2}{2}\bar{p}^T(t)(\Lambda \otimes \mathbf{I}_n)\bar{p}(t) + \left(\alpha\rho_1 + \frac{\alpha\rho_2 + \beta\rho_1}{2}\right) \lambda_{\max}(\mathbf{P})\|\bar{p}(t)\|^2 \\ &\quad + \left(\beta\rho_2 + \frac{\alpha\rho_2 + \beta\rho_1}{2}\right) \lambda_{\max}(\mathbf{P})\|\bar{q}(t)\|^2 - \frac{\beta^2}{2}\bar{q}^T(t)(\Lambda \otimes \mathbf{I}_n)\bar{q}(t) \\ &\leq -\frac{1}{2}(\bar{p}^T(t), \bar{q}^T(t))[\mathbf{R}_1 \otimes \mathbf{I}_n] \begin{pmatrix} \bar{p}(t) \\ \bar{q}(t) \end{pmatrix} \\ &\leq -\lambda_{\min}(\Theta^{-1} \mathbf{R}_1)V(t) \\ &= -\eta_1 V(t), \end{aligned} \quad (13)$$

where

$$\mathbf{R}_1 = \begin{pmatrix} \alpha^2 \Lambda - \xi_1 \mathbf{I}_{N-1} & \mathbf{O}_{N-1} \\ \mathbf{O}_{N-1} & \beta^2 \Lambda - \xi_2 \mathbf{I}_{N-1} \end{pmatrix},$$

$\eta_1 = \lambda_{\min}(\Theta^{-1} \mathbf{R}_1)$ ,  $\xi_1 = [\alpha(2\rho_1 + \rho_2) + \beta\rho_1] \lambda_{\max}(\mathbf{P})$ , and  $\xi_2 = [\beta(2\rho_2 + \rho_1) + \alpha(\rho_2 + 2)] \lambda_{\max}(\mathbf{P})$ .

For  $t \in [t_m + \delta_m, t_{m+1})$ ,  $m \in \mathbb{N}_+$ , based on Assumption 1, the time derivative of  $V(t)$  with equation (8) is

$$\begin{aligned} \dot{V}(t) &= \alpha \beta \bar{p}^T(t) (\mathbf{\Lambda} \otimes \mathbf{I}_n) \bar{q}(t) + \alpha \bar{q}^T(t) (\mathbf{P} \otimes \mathbf{I}_n) \bar{q}(t) + (\alpha \bar{p}^T(t) + \beta \bar{q}^T(t)) (\mathbf{P} \otimes \mathbf{I}_n) F(t, \bar{p}(t), \bar{q}(t)) \\ &\leq \alpha \beta \bar{p}^T(t) (\mathbf{\Lambda} \otimes \mathbf{I}_n) \bar{q}(t) + \alpha \bar{q}^T(t) (\mathbf{P} \otimes \mathbf{I}_n) \bar{q}(t) + \left( \alpha \rho_1 + \frac{\alpha \rho_2 + \beta \rho_1}{2} \right) \lambda_{\max}(\mathbf{P}) \|\bar{p}(t)\|^2 + \left( \beta \rho_2 + \frac{\alpha \rho_2 + \beta \rho_1}{2} \right) \lambda_{\max}(\mathbf{P}) \|\bar{q}(t)\|^2 \\ &\leq \frac{1}{2} (\bar{p}^T(t), \bar{q}^T(t)) (\mathbf{R}_2 \otimes \mathbf{I}_n) \begin{pmatrix} \bar{p}(t) \\ \bar{q}(t) \end{pmatrix} \\ &\leq \eta_2 V(t), \end{aligned} \quad (14)$$

where

$$\mathbf{R}_2 = \begin{pmatrix} \xi_1 \mathbf{I}_{N-1} & \alpha \beta \mathbf{\Lambda} \\ \alpha \beta \mathbf{\Lambda} & \xi_2 \mathbf{I}_{N-1} \end{pmatrix},$$

$$\eta_2 = \lambda_{\max}(\mathbf{\Theta}^{-1} \mathbf{R}_2), \xi_1 = [\alpha(2\rho_1 + \rho_2) + \beta \rho_1] \lambda_{\max}(\mathbf{P}), \text{ and } \xi_2 = [\beta(2\rho_2 + \rho_1) + \alpha(\rho_2 + 2)] \lambda_{\max}(\mathbf{P}).$$

In combination with the above process, we have

$$V(t_2) \leq V(t_1 + \delta_1) e^{\eta_2(t_2 - t_1 - \delta_1)} \leq V(t_1) e^{-\zeta_1},$$

where  $\zeta_1 = \eta_1 \delta_1 - \eta_2(t_2 - t_1 - \delta_1)$ . Applying the mentioned condition (10), one gets  $\zeta_1 > 0$ . Employing induction, it has that, for any positive integer  $m$ ,

$$V(t_{m+1}) \leq V(t_1) e^{-\sum_{j=1}^m \zeta_j},$$

where  $\zeta_j = \eta_1 \delta_j - \eta_2(t_{j+1} - t_j - \delta_j) > 0$ ,  $j = 1, 2, \dots, m$ . For arbitrary  $t > 0$ , there exists  $s \in \mathbb{N}$  such that  $t_{s+1} \leq t < t_{s+2}$ . Since  $[t_m, t_{m+1})$ ,  $m \in \mathbb{N}_+$  is non-overlapping uniformly bounded time sequence, one may let  $\omega_{\max} = \max_{m \in \mathbb{N}_+} \{t_{m+1} - t_m\}$  and  $\bar{\zeta} = \min_{j \in \mathbb{N}_+} \{\zeta_j\}$ . Consequently, it is resulting that

$$V(t) \leq V(t_{s+1}) e^{\omega_{\max} \eta_2} \leq K_0 e^{-\frac{\bar{\zeta}}{\omega_{\max}} t},$$

where  $K_0 = e^{\omega_{\max} \eta_2} V(t_0)$ , which indicates that all agents exponentially reach consensus.

### 3 Numerical simulations

In order to illustrate the effectiveness of the protocol, a simulation example is presented.

Consider the MAS (4) with five agents. The information communication among all agents is described by Fig 1 with the weight on each edge being 1. By reordering scheme, the communication graph is given as Fig 2, where the solid lines denote the directed spanning tree  $\bar{\mathbf{G}}$ . The nonlinear function is  $f(t, p_i(t), q_i(t)) = 0.05 \sin(p_i(t)) + 0.05 \cos(q_i(t))$ , where  $p_i(t), q_i(t) \in \mathbb{R}^3$ ,  $i = 1, \dots, 5$ . It is easy to verify that Assumption 1 is satisfied. According to Lemma 2, one has

$$\mathbf{K} = \begin{pmatrix} 3 & -1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & -1 \\ -2 & 1 & 0 & 1 \end{pmatrix}.$$

Choose

$$\mathbf{P} = \begin{pmatrix} 0.4417 & -0.1003 & -0.1413 & 0.2917 \\ -0.1003 & 0.5038 & -0.1608 & -0.1042 \\ -0.1413 & -0.1608 & 0.4010 & -0.0625 \\ 0.2917 & -0.1042 & -0.0625 & 0.5000 \end{pmatrix},$$

$\alpha = 2$ ,  $\beta = 3$ , then it is easy to get that  $\xi_1 = 0.3783$ ,  $\xi_2 = 3.8252$ ,  $\lambda_{\min}(\mathbf{\Lambda}) = 0.9996 > \max\{0.0946, 0.4250\}$ . By simple calculation, one obtains  $\eta_1 = 0.5637$ ,  $\eta_2 = 9.5400$ , and  $\frac{\eta_2}{\eta_2 + \eta_1} = 0.9442$ . Hence, we choose  $\frac{\delta_m}{t_{m+1} - t_m} = 0.95$ . Then the conditions of Theorem 1 are satisfied. Choosing the initial conditions  $p_1(0) = (1.2, -2, -3)^T$ ,  $q_1(0) = (-1.5, 1.3, -0.5)^T$ ,  $p_2(0) = (-3.6, -0.25, -1.5)^T$ ,  $q_2(0) = (-0.3, 0.5, -0.4)^T$ ,  $p_3(0) = (0.5, 2, -0.25)^T$ ,  $q_3(0) = (-0.6, 0.45, -0.8)^T$ ,  $p_4(0) = (-2.6, -0.8, 1.2)^T$ ,  $q_4(0) = (-2.5, 0.7, 0.5)^T$ ,  $p_5(0) = (-0.8, 1.3, -0.9)^T$ , and  $q_5(0) = (3.2, 1, 0.4)^T$ . Then, all agents' state trajectories are presented in Figs 3~Figs 8.

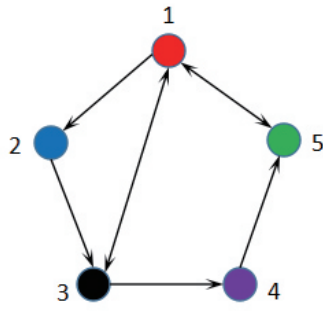


Fig 1 The graph G

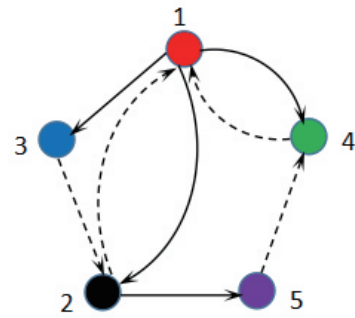


Fig 2 The reordered graph G

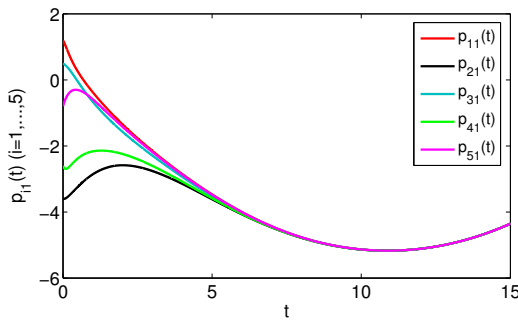


Fig 3 Trajectories of  $p_{11}$

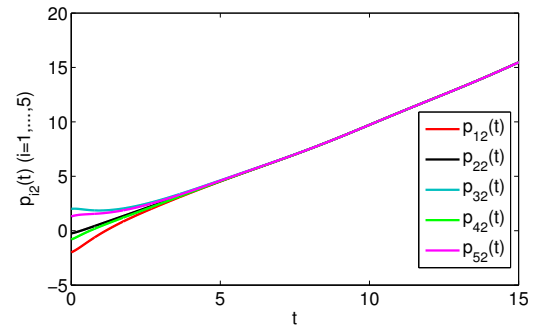


Fig 4 Trajectories of  $p_{12}$

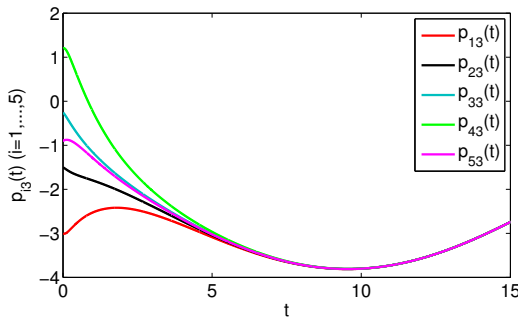


Fig 5 Trajectories of  $p_{13}$

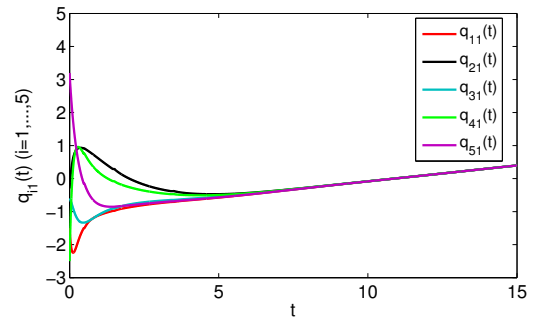


Fig 6 Trajectories of  $q_{11}$

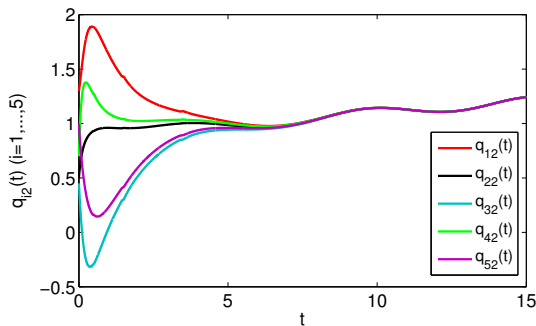


Fig 7 Trajectories of  $q_{12}$

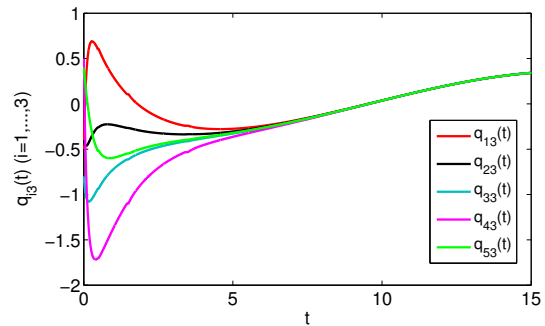


Fig 8 Trajectories of  $q_{13}$

## 4 Conclusion

In this paper, we employ a directed spanning tree transformation method to study the consensus of second-order MASs with nonlinear dynamics. A directed intermittent communication protocol is proposed, and some conditions to achieve consensus are obtained by using Lyapunov stability theory and inequality techniques.

# 有向间歇通讯下多智能体系统的一致性\*

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**摘要:** 本文研究了二阶多智能体系统在有向间歇通讯下的一致性。首先, 运用一种树型转换方法得到一致性误差系统; 其次, 利用 Lyapunov 稳定理论和矩阵不等式技巧, 得到多智能体系统依赖耦合强度和通讯宽度的一致性条件; 最后, 给出一个数值算例来验证理论结果的有效性。

**关键词:** 多智能体系统; 一致性; 有向间歇通讯

**DOI:** 10.13568/j.cnki.651094.651316.2020.04.26.002

**中图分类号:** O231.5 **文献标识码:** A **文章编号:** 2096-7675(2021)03-0257-012

**引文格式:** 于志永, 于淑珍, 蒋海军. 有向间歇通讯下多智能体系统的一致性[J]. 新疆大学学报(自然科学版)(中英文), 2021, 38(3): 257-268+284.

## 0 引言

在过去的十几年里, 多智能体系统的协同控制得到了快速的发展. 多智能体系统的协同控制包括一致性控制、会合控制、编队控制等, 一致性是该领域研究的重要问题. 由于它在交通控制、传感网络、移动机器人和无人机飞行<sup>[1-4]</sup>等领域的广泛应用而备受关注.

一致性指一组多智能体通过信息交流达到某些状态相同的行为, 例如位置、速度、姿态等. 在一致性研究中, 主要的任务是设计仅依赖智能体邻居信息的控制协议. 文献[5]是多智能体系统一致性研究的奠基性工作. 它给出了一阶多智能体系统一致性的基本框架. 已有的一致性研究可简单地分为无领导者一致性和领导者跟随一致性. 通过设计间歇饱和控制协议, 文献[6]研究了具有领导者和跟随者的半全局一致性. 文献[7]研究了间歇控制和自适应控制下多智能体系统的领导者跟随一致性. 这些研究主要考虑有向网络上的领导者跟随一致性. 然而, 许多无领导者的一致性研究主要考虑一些特殊的网路结构, 例如无向网络<sup>[8]</sup>、有向强连通网络<sup>[9]</sup>、有向平衡网络<sup>[10]</sup>等. 对于一般的有向网络, 文献[11]研究了周期间歇通讯下线性多智能体系统的一致性. 然而, 很少有研究考虑具有非周期间歇通讯和非线性动力学的二阶多智能体系统的一致性.

本文旨在提出一种新的方法研究一般有向网络上的二阶多智能体系统的一致性. 与已有的研究相比, 本文的贡献主要包括以下两点: (1) 基于节点排序方法, 本文提出了一种有向生成树转换方法. 这一方法可以将一致性问题转化为误差系统零解的稳定性问题; (2) 对于具有非线性动力学和一般有向网络拓扑的二阶多智能体系统, 本文提出了一种非周期间歇通讯的控制协议并给出二阶多智能体系统达到一致的一些准则.

## 1 预备知识和模型描述

为了方便, 我们先给出一些记号.  $R^n$  代表  $n$  维欧氏空间.  $\mathbb{N}_+$  代表正整数集.  $\mathbf{I}_N$  是  $N$  维单位矩阵.  $\mathbf{1}$  和  $\mathbf{0}$  分别代表分量全为 1 和 0 的向量.  $\mathbf{O}_N$  代表  $N$  维零矩阵. 对于矩阵  $\mathbf{C}$ ,  $C_{ij}$  代表第  $i$  行第  $j$  列的值. 对于矩阵  $\mathbf{B}$ ,  $\mathbf{B}^T$ ,  $\lambda_{\min}(\mathbf{B})$  和  $\lambda_{\max}(\mathbf{B})$  分别代表  $\mathbf{B}$  的转置, 最小特征值和最大特征值.  $\mathbf{B} > 0$  代表  $\mathbf{B}$  是一个对称正定矩阵.  $\|\cdot\|$  代表欧氏范数.  $\otimes$  代表矩阵的 Kronecker 乘积.  $\text{diag}(\cdot)$  代表对角矩阵.

考虑具有非线性动力学的二阶多智能体系统, 其中第  $i$  个智能体的动力学描述为

$$\begin{aligned} \dot{p}_i(t) &= q_i(t), \\ \dot{q}_i(t) &= f(t, p_i(t), q_i(t)) + u_i(t), \quad i = 1, 2, \dots, N \end{aligned} \quad (1)$$

\* 收稿日期: 2020-04-26

基金项目: 天山青年项目(2018Q068); 天山雪峰项目(2018XS02); 国家自然科学基金(62003289; U1703262).

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其中:  $p_i(t) \in R^n, q_i(t) \in R^n, u_i(t) \in R^n$  分别表示位置状态、速度状态和控制输入.  $f: R^+ \times R^n \times R^n \rightarrow R^n$  是一个连续可微的向量值函数, 其表示系统的非线性动力学.

**定义1**<sup>[12]</sup> 令  $M_1^N(1) = \{\tilde{M}: \tilde{M} \in R^{(N-1) \times N}\}$ , 其中矩阵  $\tilde{M}$  的第  $i$  行只有一对非零的值  $\alpha_i$  和  $-\alpha_i$ , 其余值全部为零. 记  $M_1^N(n) = \{\tilde{M} \otimes I_n: \tilde{M} \in M_1^N(1)\}$ .

**定义2**<sup>[13]</sup> 对于二阶多智能体系统 (1) 的任意初值, 如果

$$\lim_{t \rightarrow \infty} \|p_i(t) - p_j(t)\| = 0, \quad \lim_{t \rightarrow \infty} \|q_i(t) - q_j(t)\| = 0,$$

其中:  $i, j = 1, 2, \dots, N$ , 则称二阶多智能体系统 (1) 达到一致.

**假设1** 对于任意的  $p, y, q, \mu \in R^n$  及  $t \geq 0$ , 存在两个非负常数  $\rho_1$  和  $\rho_2$  使得

$$\|f(t, p, q) - f(t, y, \mu)\| \leq \rho_1 \|p - y\| + \rho_2 \|q - \mu\|.$$

**假设2** 网络通讯图  $G = (V, E, A)$  包含一个有向生成树, 并且记  $G$  的有向生成树为  $\bar{G} = (V, \bar{E}, \bar{A})$ .

**注1** 本文采用文献[12]中提出的节点排序策略. 在有向生成树  $\bar{G}$  中, 所有节点被分成三类: 根节点、茎节点和叶子节点. 在有向生成树  $\bar{G}$  中, 我们记  $v_{i_k}$  为节点  $v_{k+1}$  的父节点, 其中  $1 \leq k \leq N-1$ , 则  $(v_{i_k}, v_{k+1})$  是  $\bar{G}$  的一条有向边. 特别地, 当  $k=1$  时,  $v_{i_k} = v_1$ , 并记  $v_{i_{i_k-1}}$  为节点  $v_{i_k}$  的父节点. 则我们给出如下引理.

**引理1**<sup>[12]</sup> 假设网络通讯图  $G = (V, E, A)$  包含一个有向生成树  $\bar{G} = (V, \bar{E}, \bar{A})$ , 其中  $\bar{E} = \{(v_{i_k}, v_{k+1}) | k = 1, 2, \dots, N-1\}$ ,  $\bar{A} = [\bar{a}_{ij}]_{N \times N}$  表示加权邻接矩阵. 当  $(v_j, v_i) \in \bar{E}$  时, 矩阵  $\bar{A}$  的元素满足  $\bar{a}_{ij} > 0$ , 否则  $\bar{a}_{ij} = 0$ . 令  $H \in M_1^N(1)$  是一个  $(N-1) \times N$  维的矩阵, 其元素满足

$$H_{ij} = \begin{cases} -1, & j = i + 1, \quad i = 1, 2, \dots, N-1, \\ 1, & \bar{a}_{i+1, j} > 0, \quad i, j = 1, 2, \dots, N-1, \\ 0, & \text{其他.} \end{cases}$$

则二阶多智能体系统 (1) 达到一致当且仅当

$$\lim_{t \rightarrow \infty} \|(H \otimes I_n)p(t)\| = 0, \quad \lim_{t \rightarrow \infty} \|(H \otimes I_n)q(t)\| = 0, \tag{2}$$

其中:  $p(t) = (p_1^T(t), p_2^T(t), \dots, p_N^T(t))^T, q(t) = (q_1^T(t), q_2^T(t), \dots, q_N^T(t))^T$ .

**引理2**<sup>[12]</sup> 假设 2 成立且  $L \in R^{N \times N}$  是有向图  $G$  的 Laplacian 矩阵. 定义矩阵  $K \in R^{(N-1) \times (N-1)}$  满足  $HL = KH$ . 则矩阵  $K = HLS$  与 Laplacian 矩阵  $L$  除零特征值外其余特征值均相同. 此外, 矩阵  $S$  的元素满足

$$S_{ij} = \begin{cases} \sum_{l=1}^{i-1} H_{i-1, l} S_{lj}, & 1 \leq j < i-1, \quad i = 2, \dots, N, \\ 0, & j = i-1, \quad i = 2, \dots, N, \\ 1, & j \geq i, \quad i = 1, 2, \dots, N-1. \end{cases}$$

**引理3** 若假设 1 成立, 则对于任意的矩阵  $\Lambda > 0$ , 存在一个对称正定矩阵  $P > 0$  使得

$$P(-K) + (-K)^T P = -\Lambda. \tag{3}$$

**证明** 由于有向图  $G$  包含一个有向生成树, 则 Laplacian 矩阵  $L$  的特征值除一个零特征值外其余的特征值都具有正实部<sup>[14]</sup>. 根据引理 2, 矩阵  $K$  的所有特征值具有正实部, 所以矩阵  $-K$  是 Hurwitz 稳定的. 因此, 存在一个对称正定矩阵  $P$  满足 Lyapunov 方程(3).

## 2 主要结论

多智能体系统 (1) 在间歇通讯协议下可以表示成如下形式

$$\begin{cases} \dot{p}_i(t) = q_i(t) \\ \dot{q}_i(t) = f(t, p_i(t), q_i(t)) - \alpha \sum_{j=1}^N L_{ij} p_j(t) - \beta \sum_{j=1}^N L_{ij} q_j(t), \quad t \in [t_m, t_m + \delta_m), \\ \dot{q}_i(t) = f(t, p_i(t), q_i(t)), \quad t \in [t_m + \delta_m, t_{m+1}), \end{cases} \tag{4}$$

其中:  $\alpha$  和  $\beta$  是耦合强度,  $\delta_m$  记作第  $m$  个通讯区间内的通讯时间,  $m \in \mathbb{N}_+$  且  $t_1=0$ .

令  $\bar{p}_k(t) = p_{i_k}(t) - p_{k+1}(t)$  及  $\bar{q}_k(t) = q_{i_k}(t) - q_{k+1}(t)$  分别为位置和速度的状态误差, 其中  $1 \leq k \leq N-1$ . 则有,

$$\bar{p}(t) = (\mathbf{H} \otimes \mathbf{I}_n)p(t), \quad \bar{q}(t) = (\mathbf{H} \otimes \mathbf{I}_n)q(t) \tag{5}$$

其中:  $\bar{p}(t) = (\bar{p}_1^T(t), \bar{p}_2^T(t), \dots, \bar{p}_{N-1}^T(t))^T$ ,  $p(t) = (p_1^T(t), p_2^T(t), \dots, p_N^T(t))^T$ ,  $\bar{q}(t) = (\bar{q}_1^T(t), \bar{q}_2^T(t), \dots, \bar{q}_{N-1}^T(t))^T$ ,  $q(t) = (q_1^T(t), q_2^T(t), \dots, q_N^T(t))^T$ . 根据引理 1, 多智能体系统 (4) 达到一致当且仅当

$$\lim_{t \rightarrow \infty} \|\bar{p}(t)\| = 0, \quad \lim_{t \rightarrow \infty} \|\bar{q}(t)\| = 0. \tag{6}$$

多智能体系统 (4) 的误差动力系统可以写成

$$\begin{cases} \dot{\bar{p}}_k(t) = \bar{q}_k(t) \\ \dot{\bar{q}}_k(t) = \bar{f}(t, \bar{p}_k(t), \bar{q}_k(t)) - \alpha \sum_{j=1}^N (\mathbf{L}_{i_k, j} - \mathbf{L}_{k+1, j}) p_j(t) - \beta \sum_{j=1}^N (\mathbf{L}_{i_k, j} - \mathbf{L}_{k+1, j}) q_j(t), \quad t \in [t_m, t_m + \delta_m), \\ \dot{\bar{q}}_k(t) = \bar{f}(t, \bar{p}_k(t), \bar{q}_k(t)), \quad t \in [t_m + \delta_m, t_{m+1}), \end{cases} \tag{7}$$

其中:  $\bar{f}(t, \bar{p}_k(t), \bar{q}_k(t)) = f(t, p_{i_k}(t), q_{i_k}(t)) - f(t, p_{k+1}(t), q_{k+1}(t))$ .

令  $F(t, \bar{p}(t), \bar{q}(t)) = (\bar{f}^T(t, \bar{p}_1(t), \bar{q}_1(t)), \dots, \bar{f}^T(t, \bar{p}_{N-1}(t), \bar{q}_{N-1}(t)))^T$ , 则误差动力系统可以写成如下的向量形式

$$\begin{cases} \dot{\bar{p}}(t) = \bar{q}(t) \\ \dot{\bar{q}}(t) = F(t, \bar{p}(t), \bar{q}(t)) - \alpha(\mathbf{H}\mathbf{L} \otimes \mathbf{I}_n)p(t) - \beta(\mathbf{H}\mathbf{L} \otimes \mathbf{I}_n)q(t), \quad t \in [t_m, t_m + \delta_m), \\ \dot{\bar{q}}(t) = F(t, \bar{p}(t), \bar{q}(t)), \quad t \in [t_m + \delta_m, t_{m+1}). \end{cases} \tag{8}$$

**定理1** 若假设 1、假设 2 成立且满足下列条件

$$\lambda_{\min}(\mathbf{\Lambda}) > \max \left\{ \frac{\xi_1}{\alpha^2}, \frac{\xi_2}{\beta^2} \right\}, \tag{9}$$

$$\delta_m > \frac{\eta_2}{\eta_1 + \eta_2} (t_{m+1} - t_m), \tag{10}$$

其中:  $\mathbf{\Lambda} = \mathbf{P}\mathbf{K} + \mathbf{K}^T\mathbf{P}$ ,  $\mathbf{P} > 0$ ,  $\eta_1 = \lambda_{\min}(\mathbf{\Theta}^{-1}\mathbf{R}_1)$ ,  $\eta_2 = \lambda_{\max}(\mathbf{\Theta}^{-1}\mathbf{R}_2)$ ,  $\xi_1 = [\alpha(2\rho_1 + \rho_2) + \beta\rho_1]\lambda_{\max}(\mathbf{P})$ ,  $\xi_2 = [\beta(2\rho_2 + \rho_1) + \alpha(\rho_2 + 2)]\lambda_{\max}(\mathbf{P})$ ,  $\mathbf{R}_1 = \begin{pmatrix} \alpha^2\mathbf{\Lambda} - \xi_1\mathbf{I}_{N-1} & \mathbf{O}_{N-1} \\ \mathbf{O}_{N-1} & \beta^2\mathbf{\Lambda} - \xi_2\mathbf{I}_{N-1} \end{pmatrix}$ ,  $\mathbf{\Theta} = \begin{pmatrix} \alpha\beta\mathbf{\Lambda} & \alpha\mathbf{P} \\ \alpha\mathbf{P} & \beta\mathbf{P} \end{pmatrix}$ ,  $\mathbf{R}_2 = \begin{pmatrix} \xi_1\mathbf{I}_{N-1} & \alpha\beta\mathbf{\Lambda} \\ \alpha\beta\mathbf{\Lambda} & \xi_2\mathbf{I}_{N-1} \end{pmatrix}$ , 则二阶多智能体系统 (4) 可以达到一致.

**证明** 我们考虑如下 Lyapunov 函数

$$V(t) = \frac{1}{2}(\bar{p}^T(t), \bar{q}^T(t))(\mathbf{\Theta} \otimes \mathbf{I}_n) \begin{pmatrix} \bar{p}(t) \\ \bar{q}(t) \end{pmatrix}, \tag{11}$$

其中:  $\mathbf{\Theta} = \begin{pmatrix} \alpha\beta\mathbf{\Lambda} & \alpha\mathbf{P} \\ \alpha\mathbf{P} & \beta\mathbf{P} \end{pmatrix}$ ,  $\mathbf{\Lambda} = \mathbf{P}\mathbf{K} + \mathbf{K}^T\mathbf{P}$ ,  $\mathbf{P} > 0$ ,  $\alpha > 0$ ,  $\beta > 0$ . 基于不等式 (9), 易知  $\mathbf{\Theta} > 0$ . 则  $V(t) \geq 0$ , 且  $V(t) = 0$  当且仅当  $\bar{p}(t) = \mathbf{0}_{n(N-1)}$ ,  $\bar{q}(t) = \mathbf{0}_{n(N-1)}$ .

对于  $t \in [t_m, t_m + \delta_m)$  及  $m \in \mathbb{N}_+$ , 将  $V(t)$  沿着 (8) 式求导数. 根据引理 2 和引理 3 可得

$$\begin{aligned} \dot{V}(t) &= \alpha\beta\bar{p}^T(t)(\mathbf{\Lambda} \otimes \mathbf{I}_n)\bar{q}(t) + \alpha\bar{q}^T(t)(\mathbf{P} \otimes \mathbf{I}_n)\bar{q}(t) + [\alpha\bar{p}^T(t) + \beta\bar{q}^T(t)] \\ &\quad \times (\mathbf{P} \otimes \mathbf{I}_n)F(t, \bar{p}(t), \bar{q}(t)) - \alpha^2\bar{p}^T(t)(\mathbf{P}\mathbf{H}\mathbf{L} \otimes \mathbf{I}_n)p(t) - \alpha\beta\bar{p}^T(t)(\mathbf{P}\mathbf{H}\mathbf{L} \otimes \mathbf{I}_n)q(t) \\ &\quad - \alpha\beta\bar{q}^T(t)(\mathbf{P}\mathbf{H}\mathbf{L} \otimes \mathbf{I}_n)p(t) - \beta^2\bar{q}^T(t)(\mathbf{P}\mathbf{H}\mathbf{L} \otimes \mathbf{I}_n)q(t) \\ &= \alpha\beta\bar{p}^T(t)(\mathbf{\Lambda} \otimes \mathbf{I}_n)\bar{q}(t) + \alpha\bar{q}^T(t)(\mathbf{P} \otimes \mathbf{I}_n)\bar{q}(t) + [\alpha\bar{p}^T(t) + \beta\bar{q}^T(t)] \end{aligned}$$

$$\begin{aligned}
& \times (\mathbf{P} \otimes \mathbf{I}_n) F(t, \bar{p}(t), \bar{q}(t)) - \frac{\alpha^2}{2} \bar{p}^T(t) (\mathbf{\Lambda} \otimes \mathbf{I}_n) \bar{p}(t) - \alpha \beta \bar{p}^T(t) (\mathbf{PK} \otimes \mathbf{I}_n) \bar{q}(t) \\
& - \alpha \beta \bar{q}^T(t) (\mathbf{PK} \otimes \mathbf{I}_n) \bar{p}(t) - \frac{\beta^2}{2} \bar{q}^T(t) (\mathbf{\Lambda} \otimes \mathbf{I}_n) \bar{q}(t) \\
& = [\alpha \bar{p}^T(t) + \beta \bar{q}^T(t)] (\mathbf{P} \otimes \mathbf{I}_n) F(t, \bar{p}(t), \bar{q}(t)) - \frac{\alpha^2}{2} \bar{p}^T(t) (\mathbf{\Lambda} \otimes \mathbf{I}_n) \bar{p}(t) \\
& - \frac{\beta^2}{2} \bar{q}^T(t) (\mathbf{\Lambda} \otimes \mathbf{I}_n) \bar{q}(t) + \alpha \bar{q}^T(t) (\mathbf{P} \otimes \mathbf{I}_n) \bar{q}(t).
\end{aligned} \tag{12}$$

根据假设 1 可得

$$\begin{aligned}
& [\alpha \bar{p}^T(t) + \beta \bar{q}^T(t)] (\mathbf{P} \otimes \mathbf{I}_n) F(t, \bar{p}(t), \bar{q}(t)) \\
& \leq \lambda_{\max}(\mathbf{P}) (\alpha \|\bar{p}(t)\| + \beta \|\bar{q}(t)\|) \|F(t, \bar{p}(t), \bar{q}(t))\| \\
& \leq \lambda_{\max}(\mathbf{P}) (\alpha \|\bar{p}(t)\| + \beta \|\bar{q}(t)\|) (\rho_1 \|\bar{p}(t)\| + \rho_2 \|\bar{q}(t)\|) \\
& \leq \left( \alpha \rho_1 + \frac{\alpha \rho_2 + \beta \rho_1}{2} \right) \lambda_{\max}(\mathbf{P}) \|\bar{p}(t)\|^2 + \left( \beta \rho_2 + \frac{\alpha \rho_2 + \beta \rho_1}{2} \right) \lambda_{\max}(\mathbf{P}) \|\bar{q}(t)\|^2.
\end{aligned} \tag{13}$$

结合 (13) 式, 等式 (12) 可以写成

$$\begin{aligned}
\dot{V}(t) & \leq \alpha \bar{q}^T(t) (\mathbf{P} \otimes \mathbf{I}_n) \bar{q}(t) - \frac{\alpha^2}{2} \bar{p}^T(t) (\mathbf{\Lambda} \otimes \mathbf{I}_n) \bar{p}(t) + \left( \alpha \rho_1 + \frac{\alpha \rho_2 + \beta \rho_1}{2} \right) \lambda_{\max}(\mathbf{P}) \|\bar{p}(t)\|^2 \\
& + \left( \beta \rho_2 + \frac{\alpha \rho_2 + \beta \rho_1}{2} \right) \lambda_{\max}(\mathbf{P}) \|\bar{q}(t)\|^2 - \frac{\beta^2}{2} \bar{q}^T(t) (\mathbf{\Lambda} \otimes \mathbf{I}_n) \bar{q}(t) \\
& \leq -\frac{1}{2} (\bar{p}^T(t), \bar{q}^T(t)) [\mathbf{R}_1 \otimes \mathbf{I}_n] \begin{pmatrix} \bar{p}(t) \\ \bar{q}(t) \end{pmatrix} \\
& \leq -\lambda_{\min}(\mathbf{\Theta}^{-1} \mathbf{R}_1) V(t) \\
& = -\eta_1 V(t),
\end{aligned} \tag{14}$$

其中:

$$\mathbf{R}_1 = \begin{pmatrix} \alpha^2 \mathbf{\Lambda} - \xi_1 \mathbf{I}_{N-1} & \mathbf{O}_{N-1} \\ \mathbf{O}_{N-1} & \beta^2 \mathbf{\Lambda} - \xi_2 \mathbf{I}_{N-1} \end{pmatrix},$$

$$\eta_1 = \lambda_{\min}(\mathbf{\Theta}^{-1} \mathbf{R}_1), \xi_1 = [\alpha(2\rho_1 + \rho_2) + \beta\rho_1] \lambda_{\max}(\mathbf{P}), \xi_2 = [\beta(2\rho_2 + \rho_1) + \alpha(\rho_2 + 2)] \lambda_{\max}(\mathbf{P}).$$

对于  $t \in [t_m + \delta_m, t_{m+1})$  及  $m \in \mathbb{N}_+$ ,  $V(t)$  沿着 (8) 式的导数为

$$\begin{aligned}
\dot{V}(t) & = \alpha \beta \bar{p}^T(t) (\mathbf{\Lambda} \otimes \mathbf{I}_n) \bar{q}(t) + \alpha \bar{q}^T(t) (\mathbf{P} \otimes \mathbf{I}_n) \bar{q}(t) + (\alpha \bar{p}^T(t) + \beta \bar{q}^T(t)) (\mathbf{P} \otimes \mathbf{I}_n) F(t, \bar{p}(t), \bar{q}(t)) \\
& \leq \alpha \beta \bar{p}^T(t) (\mathbf{\Lambda} \otimes \mathbf{I}_n) \bar{q}(t) + \alpha \bar{q}^T(t) (\mathbf{P} \otimes \mathbf{I}_n) \bar{q}(t) + \left( \alpha \rho_1 + \frac{\alpha \rho_2 + \beta \rho_1}{2} \right) \lambda_{\max}(\mathbf{P}) \|\bar{p}(t)\|^2 \\
& + \left( \beta \rho_2 + \frac{\alpha \rho_2 + \beta \rho_1}{2} \right) \lambda_{\max}(\mathbf{P}) \|\bar{q}(t)\|^2 \\
& \leq \frac{1}{2} (\bar{p}^T(t), \bar{q}^T(t)) (\mathbf{R}_2 \otimes \mathbf{I}_n) \begin{pmatrix} \bar{p}(t) \\ \bar{q}(t) \end{pmatrix} \\
& \leq \eta_2 V(t),
\end{aligned} \tag{15}$$

其中:

$$\mathbf{R}_2 = \begin{pmatrix} \xi_1 \mathbf{I}_{N-1} & \alpha \beta \mathbf{\Lambda} \\ \alpha \beta \mathbf{\Lambda} & \xi_2 \mathbf{I}_{N-1} \end{pmatrix},$$

$$\eta_2 = \lambda_{\max}(\mathbf{\Theta}^{-1} \mathbf{R}_2), \xi_1 = [\alpha(2\rho_1 + \rho_2) + \beta\rho_1] \lambda_{\max}(\mathbf{P}), \xi_2 = [\beta(2\rho_2 + \rho_1) + \alpha(\rho_2 + 2)] \lambda_{\max}(\mathbf{P}).$$

结合上述的分析过程, 我们得到

$$V(t_2) \leq V(t_1 + \delta_1) e^{\eta_2(t_2 - t_1 - \delta_1)} \leq V(t_1) e^{-\zeta_1},$$

其中:  $\zeta_1 = \eta_1 \delta_1 - \eta_2(t_2 - t_1 - \delta_1)$ . 运用条件 (10), 易得  $\zeta_1 > 0$ . 利用数学归纳法可知, 对于任意的正整数  $m$ ,

$$V(t_{m+1}) \leq V(t_1) e^{-\sum_{j=1}^m \zeta_j},$$

其中:  $\zeta_j = \eta_1 \delta_j - \eta_2(t_{j+1} - t_j - \delta_j) > 0, j = 1, 2, \dots, m$ . 对于任意的  $t > 0$ , 存在  $s \in \mathbb{N}$  使得  $t_{s+1} \leq t < t_{s+2}$ . 由于  $[t_m, t_{m+1}), m \in \mathbb{N}_+$  是非空一致有界序列, 可令  $\omega_{\max} = \max_{m \in \mathbb{N}_+} \{t_{m+1} - t_m\}$  及  $\bar{\zeta} = \min_{j \in \mathbb{N}_+} \{\zeta_j\}$ . 因此, 可以得到

$$V(t) \leq V(t_{s+1}) e^{\omega_{\max} \eta_2} \leq K_0 e^{-\frac{\bar{\zeta}}{\omega_{\max}} t},$$

其中:  $K_0 = e^{\omega_{\max} \eta_2} V(t_0)$ . 即所有智能体指数渐近达到一致.

### 3 数值仿真

为了说明控制协议的有效性, 我们给出如下的数值算例.

考虑具有五个智能体的多智能体系统 (4). 智能体之间的通讯拓扑用图 1 所描述, 其中每条边的权重为 1. 通过节点排序, 通讯拓扑如图 2 所示, 其中实线代表有向生成树  $\bar{\mathbf{G}}$ . 非线性函数为  $f(t, p_i(t), q_i(t)) = 0.05 \sin(p_i(t)) + 0.05 \cos(q_i(t))$ , 其中  $p_i(t), q_i(t) \in \mathbb{R}^3, i = 1, \dots, 5$ . 容易验证假设 1 成立. 根据引理 2 可得

$$\mathbf{K} = \begin{pmatrix} 3 & -1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & -1 \\ -2 & 1 & 0 & 1 \end{pmatrix}.$$

选取

$$\mathbf{P} = \begin{pmatrix} 0.4417 & -0.1003 & -0.1413 & 0.2917 \\ -0.1003 & 0.5038 & -0.1608 & -0.1042 \\ -0.1413 & -0.1608 & 0.4010 & -0.0625 \\ 0.2917 & -0.1042 & -0.0625 & 0.5000 \end{pmatrix},$$

$\alpha = 2, \beta = 3$ , 则易得  $\xi_1 = 0.3783, \xi_2 = 3.8252$ ,  $\lambda_{\min}(\mathbf{A}) = 0.9996 > \max\{0.0946, 0.4250\}$ . 通过简单计算可知  $\eta_1 = 0.5637, \eta_2 = 9.5400, \frac{\eta_2}{\eta_2 + \eta_1} = 0.9442$ . 因此, 我们选取  $\frac{\delta_m}{t_{m+1} - t_m} = 0.95$ . 则定理 1 的条件都成立. 选取系统的初值为  $p_1(0) = (1.2, -2, -3)^T, q_1(0) = (-1.5, 1.3, -0.5)^T, p_2(0) = (-3.6, -0.25, -1.5)^T, q_2(0) = (-0.3, 0.5, -0.4)^T, p_3(0) = (0.5, 2, -0.25)^T, q_3(0) = (-0.6, 0.45, -0.8)^T, p_4(0) = (-2.6, -0.8, 1.2)^T, q_4(0) = (-2.5, 0.7, 0.5)^T, p_5(0) = (-0.8, 1.3, -0.9)^T, q_5(0) = (3.2, 1, 0.4)^T$ . 则所有智能体的状态轨线如图 3~图 8 所示.

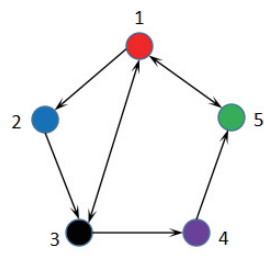


图 1 通讯图  $\mathbf{G}$

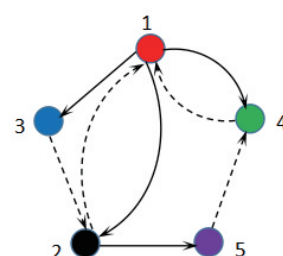


图 2 排序后的通讯图  $\mathbf{G}$

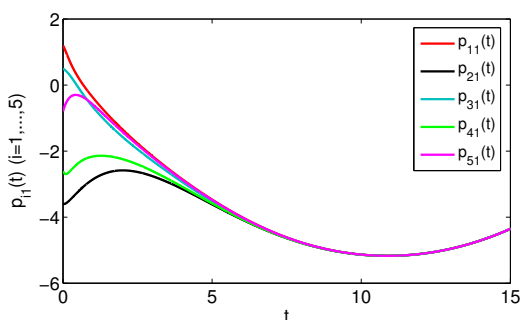


图 3  $p_{i1}$  的状态轨线

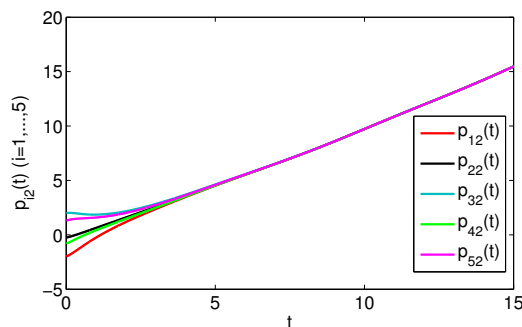
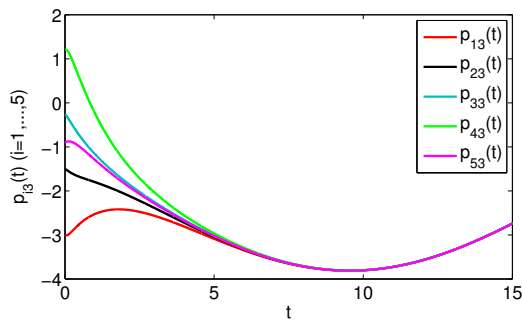
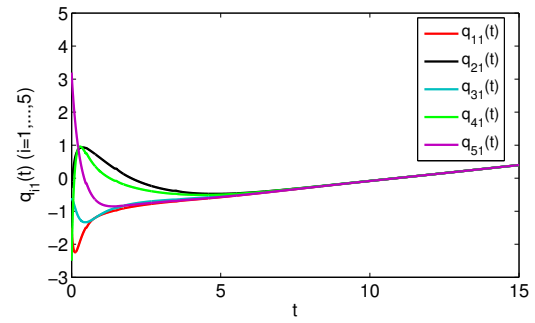
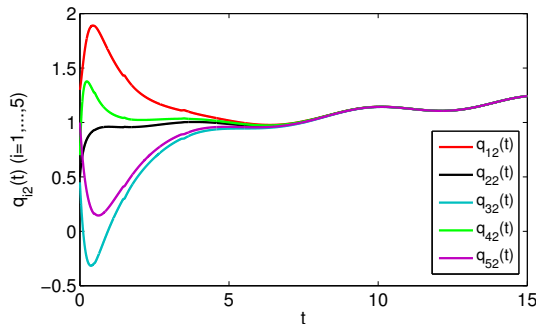
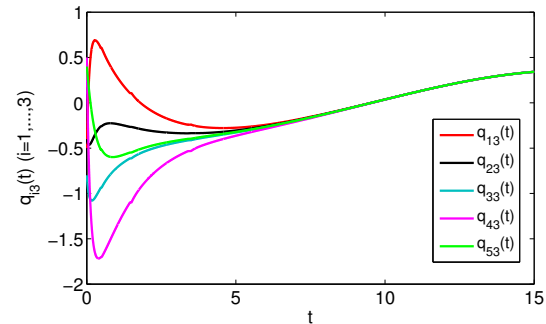


图 4  $p_{i2}$  的状态轨线

图 5  $p_{i3}$  的状态轨线图 6  $q_{i1}$  的状态轨线图 7  $q_{i2}$  的状态轨线图 8  $q_{i3}$  的状态轨线

## 4 结论

在本文中, 我们采用有向生成树转换方法来研究具有非线性动力学的二阶多智能体系统的一致性问题. 本文提出了一种有向间歇通讯协议, 并利用 Lyapunov 稳定性理论和矩阵不等式技巧得到多智能体系统达到一致的相关条件.

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