Fixed-Time Synchronization of Delayed Quaternion-Valued Neural Networks with Fractional-Order under Quantized Control*

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Abstract: The issue of fixed-time synchronization of delayed quaternion-valued neural networks (DQ-VNNs) with fractional-order under quantized control is solved. Firstly, quaternion-valued error system is separated into four real-valued subsystems, then a neoteric quantized controller is designed to reach the fixed-time synchronization goal, which can effectively reduce the transmission pressure of network. Moreover, several fixed-time synchronization criteria are derived for fractional-order DQ-VNNs, and corresponding estimates of the settling time are also yielded for fixed-time synchronization. Finally, the feasibility of theoretical results is ascertained by numerical examples.

Key words: fixed-time synchronization; quantized control; fractional-order quaternion-valued neural networks; delay

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量化控制下分数阶时滞四元值神经网络的固定时间同步

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摘　要： 解决了量化控制下分数阶时滞四元值神经网络（DQ-VNNs）的固定时间同步问题. 首先，将四元值误差系统分解为四个实值子系统，然后为实现固定时间同步目标设计了一个新的量化控制器，可以有效降低网络的传输压力. 此外，推导出了分数阶 DQ-VNNs 的几个固定时间同步准则，并对固定时间同步的停息时间作出了相应估计. 最后，通过数值例子验证了理论结果的可行性.

关键词： 固定时间同步；量化控制；分数阶四元值神经网络；时滞

0 Introduction

For recent decades, the dynamic analysis of neural networks (NNs) has drawn much attention among quantities of researchers[1−2]. Quaternion-valued NNs (Q-VNNs) have a particular advantage in efficiently dealing with multi-dimensional data compared with the traditional real-valued NNs (R-VNNs) and complex-valued NNs (C-VNNs). Taking color images as an example, the color images are made up of three primary colors in a certain proportion, where three channels $\mathfrak{i}$, $\mathfrak{j}$ and $\mathfrak{k}$ of the Q-VNNs can respectively transmit three essential colors red, green and blue. Since the switching speed of amplifiers

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is finite, time delays are widespread in NNs and may result in poor performances of the considered NNs[3]. Therefore, it is essential to study the dynamic behaviors of NNs with time delays. Fractional-order (FO) calculus is the extension of classical integer-order (IO) calculus, its history can ascend to 1695. In contrast with IO derivative, one of the characteristic advantages of FO derivative is its infinite memory capability. In view of this, it's meaningful to establish fractional-order DQ-VNNs by incorporating FO derivative into DQ-VNNs[4].

Synchronization, as a typical collective behavior, has aroused keen interest of numerous scholars on account of its crucial application prospect in many fields, including but not limited to weighted heterogeneous networks[5] and impulsive systems[6]. Compared with infinite-time synchronization, it is significant to investigate synchronization achieved in finite time due to the limited lifetime of humans and facilities. However, the settling time of finite-time synchronization (FITS) is determined by the initial conditions of considered system. In realistic applications, it's difficult or impractical to get the initial conditions in advance. In order to make up for this defect of FITS, fixed-time synchronization (FIXTS) was invented by Polyakov[7].

In light of the limited communication bandwidth in NNs, it is an efficient way to make full use of communication channels by quantizing signals before transmissions. Motivated by the aforementioned analysis, this paper focuses on the FIXTS of fractional-order DQ-VNNs by quantized control. The FIXTS criteria are gained by means of fractional calculus as well as the designed novel quantized controller, and the settling time of FIXTS is estimated.

Notations: Let $\mathbb{R}^n$ and $\mathbb{Q}^n$ be the set of n-dimensional real-valued vectors and n-dimensional quaternion-valued vectors. $\mathbb{R}$ and $\mathbb{Q}$ denote the set of real numbers as well as quaternion numbers, respectively. $\mathbb{R}^+$ represents the set of non-negative real numbers. For any $p_\alpha = p_\alpha^R + p_\alpha^I \mathfrak{i} + p_\alpha^J \mathfrak{j} + p_\alpha^K \mathfrak{k}$, where $\alpha \in \mathcal{N} = \{1, 2, \cdots, n\}$, p_α^R, p_α^I, p_α^J, $p_\alpha^K \in \mathbb{R}$, $\mathfrak{i}$, $\mathfrak{j}$ and $\mathfrak{k}$ satisfy Hamilton rules, that is $\mathfrak{i}\mathfrak{j} = -\mathfrak{j}\mathfrak{i} = \mathfrak{k}$, $\mathfrak{j}\mathfrak{k} = -\mathfrak{k}\mathfrak{j} = \mathfrak{i}$, $\mathfrak{k}\mathfrak{i} = -\mathfrak{i}\mathfrak{k} = \mathfrak{j}$ and $\mathfrak{i}^2 = \mathfrak{j}^2 = \mathfrak{k}^2 = -1$.

1 Preliminaries and model description

Definition 1[8] The Riemann-Liouville fractional integral with $\varepsilon > 0$ for function $h(t)$ is defined as

$$ {}_{t_0}I_t^{\varepsilon} h(t) = \frac{1}{\Gamma(\varepsilon)} \int_{t_0}^{t} (t-s)^{\varepsilon-1} h(s) \mathrm{d}s, $$

where $\Gamma(\cdot)$ is the Gamma function.

Definition 2[8] Caputo fractional derivative with $0 < \varepsilon < 1$ for function $h(t)$ is defined by

$$ {}_{t_0}^{c}D_t^{\varepsilon} h(t) = \frac{1}{\Gamma(1-\varepsilon)} \int_{t_0}^{t} \frac{h'(s)}{(t-s)^{\varepsilon}} \mathrm{d}s. $$

Lemma 1[9] For Caputo fractional derivation with $0 < \varepsilon < 1$, one has

$$ {}_{t_0}I_t^{\varepsilon}\, {}_{t_0}^{c}D_t^{\varepsilon} h(t) = h(t) - h(t_0). $$

Lemma 2[10] If $h(t) \in \mathbb{R}$ is a continuously differentiable function, then

$$ {}_{t_0}^{c}D_t^{\varepsilon} |h(t)| \leq \operatorname{sign}(h(t))\, {}_{t_0}^{c}D_t^{\varepsilon} h(t). $$

Lemma 3[11] Let $\vartheta_\kappa > 0$ for $\kappa \in \mathcal{N}$, then

$$ \Big(\sum_{\kappa=1}^{n} \vartheta_\kappa\Big)^{\iota} \leq C_{\mathcal{P}} \sum_{\kappa=1}^{n} \vartheta_\kappa^{\iota}, $$

where

$$ C_{\mathcal{P}} = \begin{cases} 1, & 0 < \iota \leq 1, \\ n^{\iota-1}, & \iota > 1. \end{cases} $$

Fractional-order DQ-VNNs is considered as following

$$ \begin{cases} {}_{t_0}^{c}D_t^{\varepsilon} p_\alpha(t) = -c_\alpha p_\alpha(t-\tau_1) + \sum\limits_{\beta=1}^{n} a_{\alpha\beta} h_\beta(p_\beta(t)) + \sum\limits_{\beta=1}^{n} b_{\alpha\beta} h_\beta(p_\beta(t-\tau_2)) + I_\alpha(t) \\ p_\alpha(s) = \phi_\alpha(s) \end{cases} \tag{1} $$

where $\alpha \in \mathcal{N}$, $s \in [t_0 - \tilde{\tau}, t_0]$, $0 < \varepsilon < 1$. $p_\alpha(t) \in \mathbb{Q}$, $c_\alpha \in \mathbb{R}^+$ are, respectively, state and self-regulating parameters of the αth neuron. The positive constants τ_1 and τ_2 denote the leakage and discrete delays, respectively. $\tilde{\tau} = \max\{\tau_1, \tau_2\}$. $a_{\alpha\beta}$, $b_{\alpha\beta} \in \mathbb{Q}$ represent respectively the connection weight as well as connection weight of the delayed item. $h_\beta(p_\beta(t))$, $h_\beta(p_\beta(t-\tau_2))$: $\mathbb{Q} \to \mathbb{Q}$ denote respectively activation functions of the βth neuron at t and $t-\tau_2$. $I_\alpha(t)$ is the external input.

Regard fractional-order DQ-VNNs (1) as derive system, then the revelent response system is described as

$$\begin{cases} {}_{t_0}^{c}D_t^{\varepsilon}\omega_\alpha(t) = -c_\alpha\omega_\alpha(t-\tau_1) + \sum_{\beta=1}^{n} a_{\alpha\beta}h_\beta(\omega_\beta(t)) + \sum_{\beta=1}^{n} b_{\alpha\beta}h_\beta(\omega_\beta(t-\tau_2)) + I_\alpha(t) + u_\alpha(t) \\ \omega_\alpha(s) = \psi_\alpha(s) \end{cases} \tag{2}$$

where $s \in [t_0 - \tilde{\tau}, t_0]$, $u_\alpha(t)$ is the controller to be designed for achieving FIXTS.

Assumption 1 For $\omega_\beta(t) = \omega_\beta^R(t) + \omega_\beta^I(t)\mathfrak{i} + \omega_\beta^J(t)\mathfrak{j} + \omega_\beta^K(t)\mathfrak{k} \in \mathbb{Q}$, where $\beta \in \mathcal{N}$, $\omega_\beta^R(t)$, $\omega_\beta^I(t)$, $\omega_\beta^J(t)$, $\omega_\beta^K(t) \in \mathbb{R}$, the quaternion-valued activation function $h_\beta(\omega_\beta(t))$ can be represented as

$$h_\beta(\omega_\beta(t)) = h_\beta^R(\omega_\beta^R(t)) + h_\beta^I(\omega_\beta^I(t))\mathfrak{i} + h_\beta^J(\omega_\beta^J(t))\mathfrak{j} + h_\beta^K(\omega_\beta^K(t))\mathfrak{k}.$$

Let $e_\alpha(t) = \omega_\alpha(t) - p_\alpha(t)$, combining (1) with (2) can derive error system described as follows:

$${}_{t_0}^{c}D_t^{\varepsilon}e_\alpha(t) = -c_\alpha e_\alpha(t-\tau_1) + \sum_{\beta=1}^{n} a_{\alpha\beta}[h_\beta(\omega_\beta(t)) - h_\beta(p_\beta(t))] + \sum_{\beta=1}^{n} b_{\alpha\beta}[h_\beta(\omega_\beta(t-\tau_2)) - h_\beta(p_\beta(t-\tau_2))] + u_\alpha(t) \tag{3}$$

where the initial value of error system (3) is $e_\alpha(s) = \psi_\alpha(s) - \phi_\alpha(s)$.

According to the Hamilton rules and Assumption 1, the error system (3) can be separated into the following four real-valued error systems.

$$\begin{aligned} {}_{t_0}^{c}D_t^{\varepsilon}e_\alpha^R(t) = & -c_\alpha e_\alpha^R(t-\tau_1) + \sum_{\beta=1}^{n}\Big[a_{\alpha\beta}^R\Big(h_\beta^R(\omega_\beta^R(t)) - h_\beta^R(p_\beta^R(t))\Big) - a_{\alpha\beta}^I\Big(h_\beta^I(\omega_\beta^I(t)) \\ & - h_\beta^I(p_\beta^I(t))\Big) - a_{\alpha\beta}^J\Big(h_\beta^J(\omega_\beta^J(t)) - h_\beta^J(p_\beta^J(t))\Big) - a_{\alpha\beta}^K\Big(h_\beta^K(\omega_\beta^K(t)) \\ & - h_\beta^K(p_\beta^K(t))\Big)\Big] + \sum_{\beta=1}^{n}\Big[b_{\alpha\beta}^R\Big(h_\beta^R(\omega_\beta^R(t-\tau_2)) - h_\beta^R(p_\beta^R(t-\tau_2))\Big) \\ & - b_{\alpha\beta}^I\Big(h_\beta^I(\omega_\beta^I(t-\tau_2)) - h_\beta^I(p_\beta^I(t-\tau_2))\Big) - b_{\alpha\beta}^J\Big(h_\beta^J(\omega_\beta^J(t-\tau_2)) \\ & - h_\beta^J(p_\beta^J(t-\tau_2))\Big) - b_{\alpha\beta}^K\Big(h_\beta^K(\omega_\beta^K(t-\tau_2)) - h_\beta^K(p_\beta^K(t-\tau_2))\Big)\Big] + u_\alpha^R(t) \end{aligned} \tag{4}$$

$$\begin{aligned} {}_{t_0}^{c}D_t^{\varepsilon}e_\alpha^I(t) = & -c_\alpha e_\alpha^I(t-\tau_1) + \sum_{\beta=1}^{n}\Big[a_{\alpha\beta}^I\Big(h_\beta^R(\omega_\beta^R(t)) - h_\beta^R(p_\beta^R(t))\Big) + a_{\alpha\beta}^R\Big(h_\beta^I(\omega_\beta^I(t)) \\ & - h_\beta^I(p_\beta^I(t))\Big) - a_{\alpha\beta}^K\Big(h_\beta^J(\omega_\beta^J(t)) - h_\beta^J(p_\beta^J(t))\Big) + a_{\alpha\beta}^J\Big(h_\beta^K(\omega_\beta^K(t)) \\ & - h_\beta^K(p_\beta^K(t))\Big)\Big] + \sum_{\beta=1}^{n}\Big[b_{\alpha\beta}^I\Big(h_\beta^R(\omega_\beta^R(t-\tau_2)) - h_\beta^R(p_\beta^R(t-\tau_2))\Big) \\ & + b_{\alpha\beta}^R\Big(h_\beta^I(\omega_\beta^I(t-\tau_2)) - h_\beta^I(p_\beta^I(t-\tau_2))\Big) - b_{\alpha\beta}^K\Big(h_\beta^J(\omega_\beta^J(t-\tau_2)) \\ & - h_\beta^J(p_\beta^J(t-\tau_2))\Big) + b_{\alpha\beta}^J\Big(h_\beta^K(\omega_\beta^K(t-\tau_2)) - h_\beta^K(p_\beta^K(t-\tau_2))\Big)\Big] + u_\alpha^I(t) \end{aligned} \tag{5}$$

$$\begin{aligned} {}_{t_0}^{c}D_t^{\varepsilon}e_\alpha^J(t) = & -c_\alpha e_\alpha^J(t-\tau_1) + \sum_{\beta=1}^{n}\Big[a_{\alpha\beta}^J\Big(h_\beta^R(\omega_\beta^R(t)) - h_\beta^R(p_\beta^R(t))\Big) + a_{\alpha\beta}^K\Big(h_\beta^I(\omega_\beta^I(t)) \\ & - h_\beta^I(p_\beta^I(t))\Big) + a_{\alpha\beta}^R\Big(h_\beta^J(\omega_\beta^J(t)) - h_\beta^J(p_\beta^J(t))\Big) - a_{\alpha\beta}^I\Big(h_\beta^K(\omega_\beta^K(t)) \\ & - h_\beta^K(p_\beta^K(t))\Big)\Big] + \sum_{\beta=1}^{n}\Big[b_{\alpha\beta}^J\Big(h_\beta^R(\omega_\beta^R(t-\tau_2)) - h_\beta^R(p_\beta^R(t-\tau_2))\Big) \\ & + b_{\alpha\beta}^K\Big(h_\beta^I(\omega_\beta^I(t-\tau_2)) - h_\beta^I(p_\beta^I(t-\tau_2))\Big) + b_{\alpha\beta}^R\Big(h_\beta^J(\omega_\beta^J(t-\tau_2)) \\ & - h_\beta^J(p_\beta^J(t-\tau_2))\Big) - b_{\alpha\beta}^I\Big(h_\beta^K(\omega_\beta^K(t-\tau_2)) - h_\beta^K(p_\beta^K(t-\tau_2))\Big)\Big] + u_\alpha^J(t) \end{aligned} \tag{6}$$

$$\begin{aligned}{}_{t_0}^{c}D_t^{\varepsilon}e_\alpha^K(t) &= -c_\alpha e_\alpha^K(t-\tau_1)+\sum_{\beta=1}^{n}\Big[a_{\alpha\beta}^K\Big(h_\beta^R(\omega_\beta^R(t))-h_\beta^R(p_\beta^R(t))\Big)-a_{\alpha\beta}^J\Big(h_\beta^I(\omega_\beta^I(t)) \\ &\quad -h_\beta^I(p_\beta^I(t))\Big)+a_{\alpha\beta}^I\Big(h_\beta^J(\omega_\beta^J(t))-h_\beta^J(p_\beta^J(t))\Big)+a_{\alpha\beta}^R\Big(h_\beta^K(\omega_\beta^K(t)) \\ &\quad -h_\beta^K(p_\beta^K(t))\Big)\Big]+\sum_{\beta=1}^{n}\Big[b_{\alpha\beta}^K\Big(h_\beta^R(\omega_\beta^R(t-\tau_2))-h_\beta^R(p_\beta^R(t-\tau_2))\Big) \\ &\quad -b_{\alpha\beta}^J\Big(h_\beta^I(\omega_\beta^I(t-\tau_2))-h_\beta^I(p_\beta^I(t-\tau_2))\Big)+b_{\alpha\beta}^I\Big(h_\beta^J(\omega_\beta^J(t-\tau_2)) \\ &\quad -h_\beta^J(p_\beta^J(t-\tau_2))\Big)+b_{\alpha\beta}^R\Big(h_\beta^K(\omega_\beta^K(t-\tau_2))-h_\beta^K(p_\beta^K(t-\tau_2))\Big)\Big]+u_\alpha^K(t)\end{aligned} \tag{7}$$

Assumption 2 For any $q_1, q_2 \in \mathbb{Q}$, there exists positive constant m_β satisfying

$$|h_\beta(q_1)-h_\beta(q_2)| \leq m_\beta|q_1-q_2|.$$

Lemma 4[12] If there exists C-regular function $V(\cdot):\mathbb{R}^n\to\mathbb{R}$ and constants $\delta,\pi>0$ and $\varrho>1$ such that

$$\dot{V}(e(t)) \leq -\delta-\pi V^{\varrho}(e(t)),\ e(t)\in\mathbb{R}^n\backslash\{0\} \tag{8}$$

then the origin of system can achieve fixed-time stability, and the corresponding setting time T^* is estimated by

$$T^* = \frac{1}{\delta}(\frac{\delta}{\pi})^{\frac{1}{\varrho}}(1+\frac{1}{\varrho-1}) \tag{9}$$

Definition 3 The fractional-order DQ-VNNs (1) and (2) can achieve FIXTS if for any initial value $e(t_0)$, there exists T^* (T^* has no concern with $e(t_0)$) such that

$$\lim_{t\to T^*}|e_\alpha(t)|=0,\quad |e_\alpha(t)|=0,\quad \forall t\geq T^*.$$

where T^* is called the settling time of FIXTS.

2 FIXTS criteria under quantized control

In this section, the FIXTS criteria are derived for fractional-order DQ-VNNs (1) and (2) by constructing the following novel quantized controller.

$$u_\alpha^\lambda(t) = -\varsigma_\alpha^\lambda Q(e_\alpha^\lambda(t))-\mathrm{sign}(e_\alpha^\lambda(t))\Big[\zeta_\alpha^\lambda+\mu_\alpha^\lambda|Q(e_\alpha^\lambda(t-\tau_1))|+\nu_\alpha^\lambda|Q(e_\alpha^\lambda(t-\tau_2))|+\varpi_\alpha^\lambda\Big({}_{t_0}^{c}D_t^{\varepsilon-1}|Q(e_\alpha^\lambda(t))|\Big)^{\varrho}\Big] \tag{10}$$

where $\lambda=R,I,J,K$, $\varsigma_\alpha^\lambda,\zeta_\alpha^\lambda,\mu_\alpha^\lambda,\nu_\alpha^\lambda,\varpi_\alpha^\lambda>0$, $\varrho>1$. $Q(\cdot):\mathbb{R}\to\Xi$ is the logarithmic quantizer, which is described as follows:

$$Q(l)=\begin{cases}\sigma_i, & \frac{1}{1+\xi}\sigma_i<l\leq\frac{1}{1-\xi}\sigma_i,\\ 0, & l=0,\\ -Q(-l), & l<0,\end{cases}$$

where $l\in\mathbb{R}$, $\Xi=\{\pm\sigma_i:\sigma_i=\rho^i\sigma_0, 0<\rho<1, i=0,\pm1,\cdots\}\cup\{0\}$ with $\sigma_0>0$, $\xi=\frac{1-\rho}{1+\rho}$, in light of the theory of Filippov, there exists a Filippov solution $\gamma\in[-\xi,\xi)$ satisfying $Q(l)=(1+\gamma)l$.

Theorem 1 Under Assumption 1 and quantized controller (10), fractional-order DQ-VNNs (1) and (2) can achieve FIXTS if the following three conditions are true:

(i) $c_\alpha\leq(1-\xi)\mu_\alpha^\lambda$,

(ii) $\sum\limits_{\beta=1}^{n}(|b_{\beta\alpha}^R|+|b_{\beta\alpha}^I|+|b_{\beta\alpha}^J|+|b_{\beta\alpha}^K|)m_\alpha^\lambda\leq(1-\xi)\nu_\alpha^\lambda$,

(iii) $\sum\limits_{\beta=1}^{n}(|a_{\beta\alpha}^R|+|a_{\beta\alpha}^I|+|a_{\beta\alpha}^J|+|a_{\beta\alpha}^K|)m_\alpha^\lambda\leq(1-\xi)\varsigma_\alpha^\lambda$,

where $\lambda=R,I,J,K$. Furthermore, the settling time of FIXTS is estimated as

$$T_{\max}^* = \frac{1}{\zeta}\Big(\frac{\zeta}{\varpi(1-\xi)^{\varrho}n^{1-\varrho}}\Big)^{\frac{1}{\varrho}}(1+\frac{1}{\varrho-1}) \tag{11}$$

Proof Constructing a Lyapunov function candidate as following:

$$V(e(t)) = V_1(e^R(t)) + V_2(e^I(t)) + V_3(e^J(t)) + V_4(e^K(t)) \tag{12}$$

where $V_1(e^R(t)) = \sum\limits_{\alpha=1}^{n} {}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^R(t)|$, $V_2(e^I(t)) = \sum\limits_{\alpha=1}^{n} {}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^I(t)|$, $V_3(e^J(t)) = \sum\limits_{\alpha=1}^{n} {}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^J(t)|$, $V_4(e^K(t)) = \sum\limits_{\alpha=1}^{n} {}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^K(t)|$.

According to Assumption 1 and Lemma 2, calculating the derivative of $V_1(e^R(t))$ along the trajectory (4) with quantized controller (10) has

$$\begin{aligned}
\dot{V}_1(e^R(t)) &= {}_{t_0}^{c}D_t^{\varepsilon}\Big({}_{t_0}^{c}D_t^{1-\varepsilon}V_1(e^R(t))\Big)\\
&= {}_{t_0}^{c}D_t^{\varepsilon}\Big({}_{t_0}^{c}D_t^{1-\varepsilon}\sum_{\alpha=1}^{n}{}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^R(t)|\Big)\\
&= \sum_{\alpha=1}^{n}\Big({}_{t_0}^{c}D_t^{\varepsilon}|e_\alpha^R(t)|\Big)\\
&\le \sum_{\alpha=1}^{n}\Big[\mathrm{sign}(e_\alpha^R(t)){}_{t_0}^{c}D_t^{\varepsilon}e_\alpha^R(t)\Big]\\
&\le \sum_{\alpha=1}^{n}c_\alpha|e_\alpha^R(t-\tau_1)| + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\Big[|a_{\alpha\beta}^R|m_\beta^R|e_\beta^R(t)| + |a_{\alpha\beta}^I|m_\beta^I|e_\beta^I(t)|\\
&\quad + |a_{\alpha\beta}^J|m_\beta^J|e_\beta^J(t)| + |a_{\alpha\beta}^K|m_\beta^K|e_\beta^K(t)| + |b_{\alpha\beta}^R|m_\beta^R|e_\beta^R(t-\tau_2)|\\
&\quad + |b_{\alpha\beta}^I|m_\beta^I|e_\beta^I(t-\tau_2)| + |b_{\alpha\beta}^J|m_\beta^J|e_\beta^J(t-\tau_2)| + |b_{\alpha\beta}^K|m_\beta^K|e_\beta^K(t-\tau_2)|\Big]\\
&\quad -(1-\xi)\sum_{\alpha=1}^{n}\varsigma_\alpha^R|e_\alpha^R(t)| - \sum_{\alpha=1}^{n}\zeta_\alpha^R - (1-\xi)\sum_{\alpha=1}^{n}\mu_\alpha^R|e_\alpha^R(t-\tau_1)|\\
&\quad -(1-\xi)\sum_{\alpha=1}^{n}\nu_\alpha^R|e_\alpha^R(t-\tau_2)| - \sum_{\alpha=1}^{n}\varpi_\alpha^R(1-\xi)^{\varrho}\Big({}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^R(t)|\Big)^{\varrho}
\end{aligned} \tag{13}$$

Similarly, the derivatives of $V_2(e^I(t))$, $V_3(e^J(t))$ and $V_4(e^K(t))$ can be calculated respectively

$$\begin{aligned}
\dot{V}_2(e^I(t)) &\le \sum_{\alpha=1}^{n}c_\alpha|e_\alpha^I(t-\tau_1)| + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\Big[|a_{\alpha\beta}^I|m_\beta^R|e_\beta^R(t)| + |a_{\alpha\beta}^R|m_\beta^I|e_\beta^I(t)|\\
&\quad + |a_{\alpha\beta}^K|m_\beta^J|e_\beta^J(t)| + |a_{\alpha\beta}^J|m_\beta^K|e_\beta^K(t)| + |b_{\alpha\beta}^I|m_\beta^R|e_\beta^R(t-\tau_2)|\\
&\quad + |b_{\alpha\beta}^R|m_\beta^I|e_\beta^I(t-\tau_2)| + |b_{\alpha\beta}^K|m_\beta^J|e_\beta^J(t-\tau_2)| + |b_{\alpha\beta}^J|m_\beta^K|e_\beta^K(t-\tau_2)|\Big]\\
&\quad -(1-\xi)\sum_{\alpha=1}^{n}\varsigma_\alpha^I|e_\alpha^I(t)| - \sum_{\alpha=1}^{n}\zeta_\alpha^I - (1-\xi)\sum_{\alpha=1}^{n}\mu_\alpha^I|e_\alpha^I(t-\tau_1)|\\
&\quad -(1-\xi)\sum_{\alpha=1}^{n}\nu_\alpha^I|e_\alpha^I(t-\tau_2)| - \sum_{\alpha=1}^{n}\varpi_\alpha^I(1-\xi)^{\varrho}\Big({}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^I(t)|\Big)^{\varrho}
\end{aligned} \tag{14}$$

and

$$\begin{aligned}
\dot{V}_3(e^J(t)) &\le \sum_{\alpha=1}^{n}c_\alpha|e_\alpha^J(t-\tau_1)| + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\Big[|a_{\alpha\beta}^J|m_\beta^R|e_\beta^R(t)| + |a_{\alpha\beta}^K|m_\beta^I|e_\beta^I(t)|\\
&\quad + |a_{\alpha\beta}^R|m_\beta^J|e_\beta^J(t)| + |a_{\alpha\beta}^I|m_\beta^K|e_\beta^K(t)| + |b_{\alpha\beta}^J|m_\beta^R|e_\beta^R(t-\tau_2)|\\
&\quad + |b_{\alpha\beta}^K|m_\beta^I|e_\beta^I(t-\tau_2)| + |b_{\alpha\beta}^R|m_\beta^J|e_\beta^J(t-\tau_2)| + |b_{\alpha\beta}^I|m_\beta^K|e_\beta^K(t-\tau_2)|\Big]\\
&\quad -(1-\xi)\sum_{\alpha=1}^{n}\varsigma_\alpha^J|e_\alpha^J(t)| - \sum_{\alpha=1}^{n}\zeta_\alpha^J - (1-\xi)\sum_{\alpha=1}^{n}\mu_\alpha^J|e_\alpha^J(t-\tau_1)|\\
&\quad -(1-\xi)\sum_{\alpha=1}^{n}\nu_\alpha^J|e_\alpha^J(t-\tau_2)| - \sum_{\alpha=1}^{n}\varpi_\alpha^J(1-\xi)^{\varrho}\Big({}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^J(t)|\Big)^{\varrho}
\end{aligned} \tag{15}$$

as well as

$$\begin{aligned}\dot{V}_4(e^K(t)) \leq\ & \sum_{\alpha=1}^{n} c_\alpha |e_\alpha^K(t-\tau_1)| + \sum_{\alpha=1}^{n}\sum_{\beta=1}^{n}\Big[|a_{\alpha\beta}^K| m_\beta^R |e_\beta^R(t)| + |a_{\alpha\beta}^J| m_\beta^I |e_\beta^I(t)| \\ & + |a_{\alpha\beta}^I| m_\beta^J |e_\beta^J(t)| + |a_{\alpha\beta}^R| m_\beta^K |e_\beta^K(t)| + |b_{\alpha\beta}^K| m_\beta^R |e_\beta^R(t-\tau_2)| \\ & + |b_{\alpha\beta}^J| m_\beta^I |e_\beta^I(t-\tau_2)| + |b_{\alpha\beta}^I| m_\beta^J |e_\beta^J(t-\tau_2)| + |b_{\alpha\beta}^R| m_\beta^K |e_\beta^K(t-\tau_2)|\Big] \\ & -(1-\xi)\sum_{\alpha=1}^{n} \varsigma_\alpha^K |e_\alpha^K(t)| - \sum_{\alpha=1}^{n} \zeta_\alpha^K - (1-\xi)\sum_{\alpha=1}^{n} \mu_\alpha^K |e_\alpha^K(t-\tau_1)| \\ & -(1-\xi)\sum_{\alpha=1}^{n} \nu_\alpha^K |e_\alpha^K(t-\tau_2)| - \sum_{\alpha=1}^{n} \varpi_\alpha^K (1-\xi)^{\varrho}\Big({}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^K(t)|\Big)^{\varrho}\end{aligned} \tag{16}$$

Let $\varpi = \min\{\varpi_\alpha^R, \varpi_\alpha^I, \varpi_\alpha^J, \varpi_\alpha^K\}$, then it can follow from Lemma 3 that

$$\begin{aligned}& -\sum_{\alpha=1}^{n} \varpi_\alpha^R (1-\xi)^{\varrho}\Big({}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^R(t)|\Big)^{\varrho} - \sum_{\alpha=1}^{n} \varpi_\alpha^I (1-\xi)^{\varrho}\Big({}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^I(t)|\Big)^{\varrho} \\ & -\sum_{\alpha=1}^{n} \varpi_\alpha^J (1-\xi)^{\varrho}\Big({}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^J(t)|\Big)^{\varrho} - \sum_{\alpha=1}^{n} \varpi_\alpha^K (1-\xi)^{\varrho}\Big({}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^K(t)|\Big)^{\varrho} \\ & \leq -\varpi(1-\xi)^{\varrho} n^{1-\varrho}\Big({}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^R(t)| + {}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^I(t)| + {}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^J(t)| + {}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^K(t)|\Big)^{\varrho}\end{aligned} \tag{17}$$

Combining (13)~(16) with (17) has

$$\begin{aligned}\dot{V}(e(t)) =\ & \dot{V}_1(e^R(t)) + \dot{V}_2(e^I(t)) + \dot{V}_3(e^J(t)) + \dot{V}_4(e^K(t)) \\ \leq\ & -\sum_{\alpha=1}^{n}\Big[(1-\xi)\mu_\alpha^R - c_\alpha\Big]|e_\alpha^R(t-\tau_1)| - \sum_{\alpha=1}^{n}\Big[(1-\xi)\mu_\alpha^I - c_\alpha\Big]|e_\alpha^I(t-\tau_1)| \\ & -\sum_{\alpha=1}^{n}\Big[(1-\xi)\mu_\alpha^J - c_\alpha\Big]|e_\alpha^J(t-\tau_1)| - \sum_{\alpha=1}^{n}\Big[(1-\xi)\mu_\alpha^K - c_\alpha\Big]|e_\alpha^K(t-\tau_1)| \\ & -\sum_{\alpha=1}^{n}\Big[(1-\xi)\nu_\alpha^R - \sum_{\beta=1}^{n}\Big(|b_{\beta\alpha}^R| + |b_{\beta\alpha}^I| + |b_{\beta\alpha}^J| + |b_{\beta\alpha}^K|\Big)m_\alpha^R\Big]|e_\alpha^R(t-\tau_2)| \\ & -\sum_{\alpha=1}^{n}\Big[(1-\xi)\nu_\alpha^I - \sum_{\beta=1}^{n}\Big(|b_{\beta\alpha}^R| + |b_{\beta\alpha}^I| + |b_{\beta\alpha}^J| + |b_{\beta\alpha}^K|\Big)m_\alpha^I\Big]|e_\alpha^I(t-\tau_2)| \\ & -\sum_{\alpha=1}^{n}\Big[(1-\xi)\nu_\alpha^J - \sum_{\beta=1}^{n}\Big(|b_{\beta\alpha}^R| + |b_{\beta\alpha}^I| + |b_{\beta\alpha}^J| + |b_{\beta\alpha}^K|\Big)m_\alpha^J\Big]|e_\alpha^J(t-\tau_2)| \\ & -\sum_{\alpha=1}^{n}\Big[(1-\xi)\nu_\alpha^K - \sum_{\beta=1}^{n}\Big(|b_{\beta\alpha}^R| + |b_{\beta\alpha}^I| + |b_{\beta\alpha}^J| + |b_{\beta\alpha}^K|\Big)m_\alpha^K\Big]|e_\alpha^K(t-\tau_2)| \\ & -\sum_{\alpha=1}^{n}\Big[(1-\xi)\varsigma_\alpha^R - \sum_{\beta=1}^{n}\Big(|a_{\beta\alpha}^R| + |a_{\beta\alpha}^I| + |a_{\beta\alpha}^J| + |a_{\beta\alpha}^K|\Big)m_\alpha^R\Big]|e_\alpha^R(t)| \\ & -\sum_{\alpha=1}^{n}\Big[(1-\xi)\varsigma_\alpha^I - \sum_{\beta=1}^{n}\Big(|a_{\beta\alpha}^R| + |a_{\beta\alpha}^I| + |a_{\beta\alpha}^J| + |a_{\beta\alpha}^K|\Big)m_\alpha^I\Big]|e_\alpha^I(t)| \\ & -\sum_{\alpha=1}^{n}\Big[(1-\xi)\varsigma_\alpha^J - \sum_{\beta=1}^{n}\Big(|a_{\beta\alpha}^R| + |a_{\beta\alpha}^I| + |a_{\beta\alpha}^J| + |a_{\beta\alpha}^K|\Big)m_\alpha^J\Big]|e_\alpha^J(t)| \\ & -\sum_{\alpha=1}^{n}\Big[(1-\xi)\varsigma_\alpha^K - \sum_{\beta=1}^{n}\Big(|a_{\beta\alpha}^R| + |a_{\beta\alpha}^I| + |a_{\beta\alpha}^J| + |a_{\beta\alpha}^K|\Big)m_\alpha^K\Big]|e_\alpha^K(t)| \\ & -\sum_{\alpha=1}^{n}\Big[\zeta_\alpha^R + \zeta_\alpha^I + \zeta_\alpha^J + \zeta_\alpha^K\Big] - \varpi(1-\xi)^{\varrho} n^{1-\varrho}\Big(\sum_{\alpha=1}^{n} {}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^R(t)| \\ & + \sum_{\alpha=1}^{n} {}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^I(t)| + \sum_{\alpha=1}^{n} {}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^J(t)| + \sum_{\alpha=1}^{n} {}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^K(t)|\Big)^{\varrho}\end{aligned} \tag{18}$$

Let $\zeta = \sum_{\alpha=1}^{n}\left[\zeta_\alpha^R + \zeta_\alpha^I + \zeta_\alpha^J + \zeta_\alpha^K\right]$, then combining (18) with conditions (i)~(iii) yields

$$\dot{V}(e(t)) \leq -\zeta - \varpi(1-\xi)^{\varrho} n^{1-\varrho} V^{\varrho}(e(t)) \tag{19}$$

On the basis of Lemma 4, fractional-order DQ-VNNs (1) and (2) can achieve FIXTS, the settling time of FIXTS is $T^*_{\max}$.

Remark 1 The FIXTS of complex networks was studied in [13]. In [14], the FIXTS of Q-VNNs was explored. In [15], the FIXTS of FONNs was investigated. However, the FIXTS of fractional-order DQ-VNNs has not been considered until now. The FIXTS of fractional-order DQ-VNNs (1) and (2) are firstly investigated in this paper via quantized control. It should be worth pointed that our fractional-order DQ-VNNs contain the merits of Q-VNNs, FO derivative as well as leakage and discrete delays. Hence our FIXTS results are more general than matching ones derived in [13-15].

Remark 2 The quantized controller (10) is firstly designed to realize FIXTS of fractional-order DQ-VNNs. Compared with the existing FIXTS controllers for achieving FIXTS goal in [14-15], the logarithmic quantizer is taken into account in our novel controller (10), which can make full use of communication channels and effectively reduce control costs.

Remark 3 If the leakage and discrete delays are not taken into account, i.e. $\tau_1 = \tau_2 = 0$, then the fractional-order DQ-VNNs (1) can degrade into

$$\begin{cases} {}_{t_0}^{c}D_t^{\varepsilon} p_\alpha(t) = -c_\alpha p_\alpha(t) + \sum_{\beta=1}^{n} a_{\alpha\beta} h_\beta(p_\beta(t)) + I_\alpha(t) \\ p_\alpha(s) = \phi_\alpha(s) \end{cases} \tag{20}$$

meanwhile, the fractional-order DQ-VNNs (2) degenerates into

$$\begin{cases} {}_{t_0}^{c}D_t^{\varepsilon} \omega_\alpha(t) = -c_\alpha \omega_\alpha(t) + \sum_{\beta=1}^{n} a_{\alpha\beta} h_\beta(\omega_\beta(t)) + I_\alpha(t) + \hat{u}_\alpha(t) \\ \omega_\alpha(s) = \psi_\alpha(s) \end{cases} \tag{21}$$

The quantized controller (10) is modified slightly as follows:

$$\hat{u}_\alpha^\lambda(t) = -\varsigma_\alpha^\lambda Q(e_\alpha^\lambda(t)) - \operatorname{sign}(e_\alpha^\lambda(t))\left[\zeta_\alpha^\lambda + \varpi_\alpha^\lambda\left({}_{t_0}^{c}D_t^{\varepsilon-1}|Q(e_\alpha^\lambda(t))|\right)^{\varrho}\right] \tag{22}$$

Corollary 1 Let Assumption 1 and quantized controller (22) hold, if the self-regulating parameter satisfies

$$c_\alpha + (1-\xi)\varsigma_\alpha^\lambda \geq \sum_{\beta=1}^{n}(|a_{\beta\alpha}^R| + |a_{\beta\alpha}^I| + |a_{\beta\alpha}^J| + |a_{\beta\alpha}^K|)m_\alpha^\lambda \tag{23}$$

where $\alpha \in \mathcal{N}$, $\lambda = R, I, J, K$, then fractional-order DQ-VNNs (20) and (21) can achieve FIXTS within $T^{1*}_{\max} = \frac{1}{\zeta}\left(\frac{\zeta}{\varpi(1-\xi)^{\varrho} n^{1-\varrho}}\right)^{\frac{1}{\varrho}}(1 + \frac{1}{\varrho-1})$.

Remark 4 If quantized control is not considered in (10), then quantized controller (10) degenerates into

$$\breve{u}_\alpha^\lambda(t) = -\varsigma_\alpha^\lambda e_\alpha^\lambda(t) - \operatorname{sign}(e_\alpha^\lambda(t))\left[\zeta_\alpha^\lambda + \mu_\alpha^\lambda|e_\alpha^\lambda(t-\tau_1)| + \nu_\alpha^\lambda|e_\alpha^\lambda(t-\tau_2)| + \varpi_\alpha^\lambda\left({}_{t_0}^{c}D_t^{\varepsilon-1}|e_\alpha^\lambda(t)|\right)^{\varrho}\right] \tag{24}$$

Corollary 2 Let Assumption 1 and controller (24) hold, if the following three conditions are satisfied

(i) $\mu_\alpha^\lambda \geq c_\alpha$,

(ii) $\nu_\alpha^\lambda \geq \sum_{\beta=1}^{n}(|b_{\beta\alpha}^R| + |b_{\beta\alpha}^I| + |b_{\beta\alpha}^J| + |b_{\beta\alpha}^K|)m_\alpha^\lambda$,

(iii) $\varsigma_\alpha^\lambda \geq \sum_{\beta=1}^{n}(|a_{\beta\alpha}^R| + |a_{\beta\alpha}^I| + |a_{\beta\alpha}^J| + |a_{\beta\alpha}^K|)m_\alpha^\lambda$,

where $\alpha \in \mathcal{N}$, $\lambda = R, I, J, K$, then the fractional-order DQ-VNNs (1) and (2) can realize FIXTS within $T^{2*}_{\max} = \frac{1}{\zeta}(\frac{\zeta}{\varpi n^{1-\varrho}})^{\frac{1}{\varrho}}(1 + \frac{1}{\varrho-1})$.

Remark 5 When ε=1, the results of Theorem 1, Corollaries 1 and 2 are also true.

3 Numerical simulations

A numerical example is provided to verify the effectiveness of theoretical results for FIXTS in this section.

A two-dimensional fractional-order DQ-VNN is considered as follows

$$\begin{cases} {}_{t_0}^{c}D_t^{0.9}p_\alpha(t) &= -c_\alpha p_\alpha(t-\tau_1)+\sum\limits_{\beta=1}^{2}a_{\alpha\beta}h_\beta(p_\beta(t))+\sum\limits_{\beta=1}^{2}b_{\alpha\beta}h_\beta(p_\beta(t-\tau_2))+I_\alpha(t) \\ p_\alpha(s) &= \phi_\alpha(s) \end{cases} \tag{25}$$

where $\alpha=1,2$, $(p_1(t),p_2(t))^T\in\mathbb{Q}^2$, $h_\beta(p_\beta(t))=\tanh(p_\beta^R(t))+\tanh(p_\beta^I(t))\mathfrak{i}+\tanh(p_\beta^J(t))\mathfrak{j}+\tanh(p_\beta^K(t))\mathfrak{k}$ with $p_\beta^R(t),p_\beta^I(t),p_\beta^J(t),p_\beta^K(t)\in\mathbb{R}$, $I_1(t)=I_2(t)=0$, $a_{11}=0.16-0.02\mathfrak{i}+0.01\mathfrak{j}+0.05\mathfrak{k}$, $a_{12}=-0.14+0.01\mathfrak{i}+0.02\mathfrak{j}+0.03\mathfrak{k}$, $a_{21}=0.13-0.05\mathfrak{i}+0.06\mathfrak{j}+0.04\mathfrak{k}$, $a_{22}=0.14+0.01\mathfrak{i}+0.02\mathfrak{j}+0.03\mathfrak{k}$, $b_{11}=0.13-0.05\mathfrak{i}+0.06\mathfrak{j}+0.04\mathfrak{k}$, $b_{12}=-0.16+0.02\mathfrak{i}+0.01\mathfrak{j}+0.03\mathfrak{k}$, $b_{21}=0.14-0.06\mathfrak{i}+0.05\mathfrak{j}+0.02\mathfrak{k}$, $b_{22}=0.15+0.02\mathfrak{i}+0.03\mathfrak{j}+0.01\mathfrak{k}$, $c_1=c_2=0.1$, $\tau_1=3$, $\tau_2=4$, the initial value of (25) is chosen as $p_\alpha(s)=(1.6-0.7\mathfrak{i}+0.7\mathfrak{j}+0.8\mathfrak{k},-0.1+0.9\mathfrak{i}-1.9\mathfrak{j}+1.2\mathfrak{k})^T$.

The response system of (25) is described as

$$\begin{cases} {}_{t_0}^{c}D_t^{\varepsilon}\omega^{0.9}(t) &= -c_\alpha\omega_\alpha(t-\tau_1)+\sum\limits_{\beta=1}^{2}a_{\alpha\beta}h_\beta(\omega_\beta(t))+\sum\limits_{\beta=1}^{2}b_{\alpha\beta}h_\beta(\omega_\beta(t-\tau_2))+I_\alpha(t)+u_\alpha(t) \\ \omega_\alpha(s) &= \psi_\alpha(s) \end{cases} \tag{26}$$

where $\alpha=1,2$, $(\omega_1(t),\omega_2(t))^T\in\mathbb{Q}^2$, $h_\beta(\omega_\beta(t))=\tanh(\omega_\beta^R(t))+\tanh(\omega_\beta^I(t))\mathfrak{i}+\tanh(\omega_\beta^J(t))\mathfrak{j}+\tanh(\omega_\beta^K(t))\mathfrak{k}$ with $\omega_\beta^R(t),\omega_\beta^I(t),\omega_\beta^J(t)$, $\omega_\beta^K(t)\in\mathbb{R}$, the initial value of (26) is selected as $\omega_\alpha(s)=(1.6-3.7\mathfrak{i}+5.7\mathfrak{j}+2.8\mathfrak{k},-3.5+3.9\mathfrak{i}-0.9\mathfrak{j}+0.2\mathfrak{k})^T$, the other parameters of fractional-order DQ-VNNs (26) are the same as those of fractional-order DQ-VNNs (25). Choose the parameters of (10) as $\rho=0.6$, $\varsigma_\alpha^\lambda=0.7$, $\zeta_\alpha^\lambda=0.5$, $\mu_\alpha^\lambda=1.8$, $\nu_\alpha^\lambda=0.75$, $\varpi_\alpha^\lambda=0.01$, $\varrho=1.000\,1$, $\alpha=1,2$, $\lambda=R,I,J,K$.

By simple calculation, one has

$$0.1 = \max\{c_1,c_2\}\le\min_{\alpha=1,2}\{(1-\xi)\mu_\alpha^\lambda,\lambda=R,I,J,K\}=1.35,$$

$$\begin{aligned} 0.55 &= \max\{\sum_{\beta=1}^{2}(|b_{\beta\alpha}^R|+|b_{\beta\alpha}^I|+|b_{\beta\alpha}^J|+|b_{\beta\alpha}^K|)m_\alpha^\lambda,\lambda=R,I,J,K\} \\ &\le \min\{(1-\xi)\nu_\alpha^\lambda,\alpha=1,2,\lambda=R,I,J,K\} \\ &= 0.562\,5, \end{aligned}$$

$$\begin{aligned} 0.52 &= \max\{\sum_{\beta=1}^{2}(|a_{\beta\alpha}^R|+|a_{\beta\alpha}^I|+|a_{\beta\alpha}^J|+|a_{\beta\alpha}^K|)m_\alpha^\lambda,\lambda=R,I,J,K\} \\ &\le \min\{(1-\xi)\varsigma_\alpha^\lambda,\alpha=1,2,\lambda=R,I,J,K\} \\ &= 0.525. \end{aligned}$$

thereby the conditions of Theorem 1 are satisfied, the fractional-order DQ-VNNs (25) and (26) can achieve FIXTS. The time response curves of $e_\alpha(t)$ and $Q(e_\alpha(t))$ under quantized controller (10) are shown in Fig 1 and Fig 2.

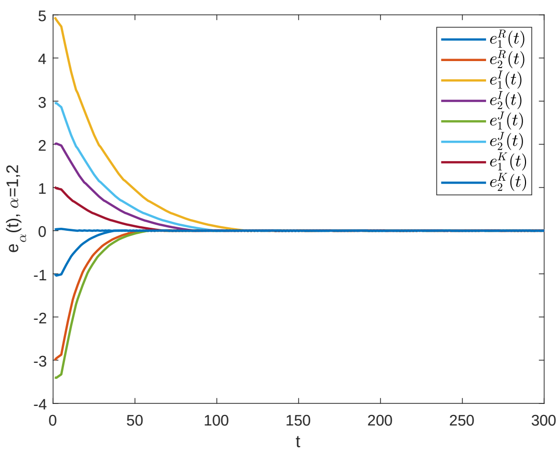

Fig 1 Time response curves of $e_\alpha(t)$, $\alpha=1,2$

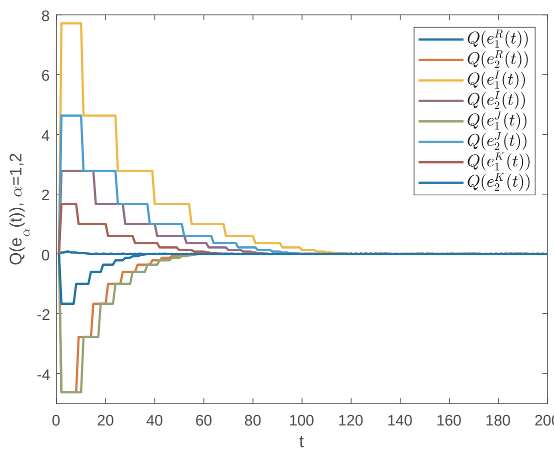

Fig 2 Time response curves of $Q(e_\alpha(t))$, $\alpha=1,2$

4 Conclusion

The FIXTS of fractional-order DQ-VNNs was analysed based on quantized control in this paper. By designing a novel quantized controller, The FIXTS criteria were derived with the help of fractional calculus theorem and inequality techniques, and the settling time of FIXTS was estimated. Since parameter uncertainties are unavoidable in engineering, consequently, it is meaningful to investigate the FIXTS of Q-VNNs with parameter uncertainties, which will be researched in our future work.

References:

[1] MUHAMMADHAJI A, LI H L. General decay synchronization for recurrent neural networks with distributed time delays[J]. Applied Mathematics and Mechanics, 2019, 40(11): 1204-1213.

[2] FENG L, HU C, YU J. Fixed-time synchronization of coupled memristive complex-valued neural networks[J]. Journal of Xinjiang University(Natural Science Edition in Chinese and English), 2021, 38(2): 129-143.

[3] LI H L, HU C, CAO J D, et al. Quasi-projective and complete synchronization of fractional-order complex-valued neural networks with time delays[J]. Neural Networks, 2019, 118: 102-109.

[4] CHEN S L, LI H L, KAO Y G, et al. Finite-time stabilization of fractional-order fuzzy quaternion-valued BAM neural networks via direct quaternion approach[J]. Journal of the Franklin Institute, 2021, 358(15): 7650-7673.

[5] 罗续鹏, 蒋海军, 陈珊珊. 加权异质网络上具有多语言环境的谣言传播建模[J]. 新疆大学学报(自然科学版)(中英文), 2022, 39(4): 393-400.

[6] LI H L, ZHANG L, WAN H Y. A periodic single species model with dissymmetric bidirectional impulse diffusion between two patches[J]. Journal of Xinjiang University(Natural Science Edition), 2014, 31(1): 70-75.

[7] POLYAKOV A. Nonlinear feedback design for fixed-time stabilization of linear control systems[J]. IEEE Transactions on Automatic Control, 2012, 57(8): 2106-2110.

[8] KILBAS A, SRIVASTAVA H, TRUJILLO J. Theory and application of fractional differential equations[M]. New York: Elsevier, 2006.

[9] PODLUBNY I. Fractional differential equations[M]. San Diego: Academic Press, 1999.

[10] CAO Y, SAMIDURAI R, SRIRAMAN R. Robust passivity analysis for uncertain neural networks with leakage delay and additive time-varying delays by using general activation function[J]. Mathematics and Computers in Simulation, 2017, 155: 57-77.

[11] KUCZMA M. An introduction to the theory of functional equations and inequalities: Cauchy's equation and Jensen's inequality[M]. Boston: Birkhäuser, 2009.

[12] HU C, YU J, CHEN Z H, et al. Fixed-time stability of dynamical systems and fixed-time synchronization of coupled discontinuous neural networks[J]. Neural Networks, 2017, 89: 74-83.

[13] ZHANG W L, LI H F, LI C D, et al. Fixed-time synchronization criteria for complex networks via quantized pinning control[J]. ISA Transactions, 2019, 91: 151-156.

[14] DENG H, BAO H B. Fixed-time synchronization of quaternion-valued neural networks[J]. Physica A: Statistical Mechanics and Its Applications, 2019, 527: 121351.

[15] WANG W P, JIA X, WANG Z, et al. Fixed-time synchronization of fractional order memristive MAM neural networks by sliding mode control[J]. Neurocomputing, 2020, 401: 364-376.

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