Prescribed-Time Leader-Follower Consensus for Nonlinear Multi-Agent Systems*

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Abstract: The prescribed-time leader-follower consensus problems are investigated for nonlinear multi-agent systems (MASs) with undirected and directed network topologies. First of all, a novel distributed control protocol is proposed for MASs with an undirected network topology. Based on Lyapunov stability theory, some sufficient conditions are derived for MASs to achieve leader-follower consensus within a prescribed-time. Moreover, we consider the directed network topology and give some sufficient conditions to ensure the prescribed-time leader-follower consensus. Finally, two examples are given to demonstrate the effectiveness of the main results.

Key words: prescribed-time; multi-agent systems; leader-follower consensus

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非线性多智能体系统的指定时间领导跟随一致性

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摘　要： 研究了非线性多智能体系统在无向和有向网络拓扑下的指定时间领导跟随一致性问题. 首先, 在无向网络拓扑下提出一种新的分布式控制协议, 运用 Lyapunov 稳定性理论, 给出多智能体系统实现指定时间领导跟随一致的相关条件. 其次, 考虑有向网络拓扑的情况, 并给出系统在指定时间内实现领导跟随一致的条件. 最后, 通过两个例子来验证理论结果的有效性.

关键词： 指定时间; 多智能体系统; 领导跟随一致性

0 Introduction

During the past decades, due to its wide applications in sensor networks, electrical power grids, and multi-unmanned air vehicle formation[1−3], the distributed cooperative control of MASs has witnessed substantial progress. In comparison with traditional centralized control, it has stronger adaptivity, higher robustness, and flexibility. As a fundamental and crucial issue of distributed cooperative control, the consensus of MASs has received extensive attention[4].

The existing consensus problems can be classified into leaderless consensus and leader-follower consensus. Compared with leaderless consensus, the leader-follower consensus can conserves resources and boosts efficiency. Therefore, the leader-

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follower consensus problems were investigated in [5-6]. However, the aforementioned consensus protocols are asymptotic convergent. That means the convergence time approaches infinity, which may not be verifiable in many practical situations. In view of this phenomenon, some researchers have developed finite-time algorithms[7−9], which can provide a faster convergence speed and better disturbance rejection performance. In [7-8], the finite-time consensus problems were put forward for first-order and second-order integrator MASs, respectively. Further, the finite-time consensus control algorithms for higher-order integrator MASs were proposed in [9]. However, the convergence time of the aforementioned results [7-9] depend on the initial conditions, which are unprocurable in real applications since the initial state is usually unknown or unavailable.

To resolve the above limitation of finite-time control, lots of results about fixed-time consensus problems have been reported[10−13]. For instance, some fixed-time consensus algorithms were proposed for first-order integrator systems[10], second-order integrator systems[11−12], and high-order integrator systems[13]. However, for the fixed-time consensus algorithms, the designer still cannot arbitrarily assign the settling time since it always depends on design parameters. In order to solve the shortcomings of the finite-time/fixed-time consensus algorithms. Wang et al.[14] proposed a novel scaling function to study the prescribed-time consensus problem of MASs. Further, a prescribed-time consensus problem was researched with second-order integrator MASs in [15]. It is worth noticing that the systems considered in [10-15] are integral dynamics. Nevertheless, in many practical applications, the nonlinear dynamics are usually used to describe system behaviors. Thus, the prescribed-time consensus for nonlinear MASs is worth considering.

Motivated by the above observations, we will study the prescribed-time leader-follower consensus problems for nonlinear MASs under undirected and directed network topologies in this paper. The major contributions of this research are as follows:

(i) In contrast to [14], the model we considered has a nonlinear function term, and the state of the leader is time-varying.

(ii) Different from the finite-time/fixed-time consensus convergence conditions, the settling time of the prescribed-time algorithms is independent of the initial state and control parameters.

Notations In this paper, $\mathcal{R}$ and $\mathcal{R}^n$ denote the set of real number and the n-dimensional Euclidean space, respectively. $I_n \in \mathcal{R}^{n\times n}$ represents $n\times n$ identity matrix. 1_n (or 0_n) denotes an n-dimensional column vector whose all entries being 1 (or 0). For a matrix A, A^T, $\lambda_{\max}(A)$, $\lambda_{\min}(A)$ represent the transpose, maximum and minimum eigenvalues of A, respectively. $A > 0$ means that A is a positive definite matrix. $\|\cdot\|$ and $\otimes$ represent the Euclidean norm and the Kronecker product, respectively.

1 Preliminaries

1.1 Graph theory

The network of MASs is usually modeled by a graph $\mathcal{G} = (\mathcal{V},\mathcal{E},\mathcal{A})$, where $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$ is the vertex set and $\mathcal{E} = \{(v_i, v_j)|v_i, v_j \in \mathcal{V}\}$ is the edge set. $(v_i, v_j) \in \mathcal{E}$ indicates a directed edge from v_i to v_j. The adjacency matrix of $\mathcal{G}$ is $\mathcal{A} = [a_{ij}] \in \mathcal{R}^{N\times N}$, $a_{ii} = 0$, $a_{ij} \neq 0 \Leftrightarrow (v_j, v_i) \in \mathcal{E}$ and $a_{ij} = 0$, otherwise. The in-degree matrix associated with $\mathcal{G}$ is defined as $\mathcal{D} = \mathrm{diag}(d_1, d_2, \cdots, d_N)$, where $d_i = \sum_{j=1}^N a_{ij}$. The Laplacian matrix is defined as $\mathcal{L} = [l_{ij}]_{N\times N}$ with $l_{ij} = -a_{ij}$ for $i \neq j$, and $l_{ii} = \sum_{j=1}^N a_{ij}$.

In this paper, the interaction graph among N followers and the leader is denoted as $\widetilde{\mathcal{G}}$. The communication subgraph among followers is defined as $\mathcal{G}$. Let matrix $\mathcal{B} = \mathrm{diag}(b_1, b_2, \cdots, b_N)$ with $b_i \in \{0, 1\}$, where $b_i = 1$ indicates that agent i receives the information from the leader, otherwise $b_i = 0$. Moreover, the matrix $\mathcal{H}$ is defined by $\mathcal{H} = \mathcal{L} + \mathcal{B}$.

1.2 Problem statement

We consider a system with one leader and N followers. The dynamics of the ith follower is given by

$$\dot{x}_i(t) = \mathcal{W} x_i(t) + f(t, x_i(t)) + u_i(t),\ i = 1, 2, \cdots N \tag{1}$$

where $x_i(t) \in \mathcal{R}^n, u_i(t) \in \mathcal{R}^n$ and $f(t, x_i(t))$ denote the state, control input and the nonlinear function of the ith follower, respectively. $\mathcal{W} \in \mathcal{R}^{n\times n}$ is a constant matrix. The leader has the following dynamics

$$\dot{x}_0(t) = \mathcal{W} x_0(t) + f(t, x_0(t)) \tag{2}$$

where $x_0(t) \in \mathcal{R}^n, f(t, x_0(t))$ represent the state and nonlinear function of the leader, respectively. Denote $x(t) = (x_1^T(t), x_2^T(t), \cdots, x_N^T(t))^T$ and $\mathcal{F}(t, x(t)) = (f^T(t, x_1(t)), f^T(t, x_2(t)), \cdots, f^T(t, x_N(t)))^T$.

Consider the system

$$\dot{\xi}(t) = g(\xi(t), t),\ \xi(0) = \xi_0,\ t \geq 0 \tag{3}$$

where $\xi(t) \in \mathcal{R}^n$ is the state vector, $g : \mathcal{R}^n \times \mathcal{R}^+ \to \mathcal{R}^n$ is a nonlinear vector valued function and $g(0, t) = 0$ for all $t > 0$.

Definition 1[16] The origin of the system (3) is said to be globally prescribed-time stable if $\lim_{t\to T} \xi(t) = 0$, $\xi(t) \equiv 0$ ($t \geq T$) and the settling time T is a user-assignable finite constant, i.e., $0 < T < \infty$, $\forall \xi_0 \in \mathcal{R}^n$.

We introduce a time-varying scaling function[14]:

$$\mu(t) = \begin{cases} \frac{T^h}{(T-t)^h}, & t \in [0, T) \\ 1, & t \in [T, \infty) \end{cases} \tag{4}$$

where $h > 0$ and $T > 0$. Note that $\mu(t)^{-r} (r > 0)$ is monotonically decreasing during the interval $[0, T)$, $\mu(0)^{-r} = 1$ and $\lim_{t\to T^-} \mu(t)^{-r} = 0$.

Lemma 1 Let $V(\xi(t)) : \mathcal{R}^n \to \mathcal{R}$ is a continuous and non-negative function. If there exist $\alpha > 0$, $\beta > 0$ such that

$$\dot{V}(\xi(t)) \leq -\alpha \frac{\dot{\mu}(t)}{\mu(t)} V(\xi(t)) - \beta V(\xi(t)), \quad t \in [0, T) \tag{5}$$

$$\dot{V}(\xi(t)) \leq 0, \quad t \in [T, \infty) \tag{6}$$

then the origin of the system (3) is globally prescribed-time stable with the settling time T. Additionally,

$$V(\xi(t)) \leq \mu^{-\alpha}(t) \exp(-\beta t) V(\xi(0)), \quad t \in [0, T) \tag{7}$$

and $V(\xi(t)) \equiv 0$ for any $t \in [T, \infty)$.

Proof When $t \in [0, T)$, by multiplying $\mu^\alpha(t)$ on both sides of (5), one has

$$\mu^\alpha(t) \dot{V}(\xi(t)) \leq -\alpha \dot{\mu}(t) \mu(t)^{\alpha-1} V(\xi(t)) - \beta \mu^\alpha(t) V(\xi(t)).$$

Then, it follows that

$$\frac{\mathrm{d}(\mu^\alpha(t) V(\xi(t)))}{\mathrm{d}t} = \alpha \dot{\mu}(t) \mu(t)^{\alpha-1} V(\xi(t)) + \mu^\alpha(t) \dot{V}(\xi(t)) \leq -\beta \mu^\alpha(t) V(\xi(t)) \tag{8}$$

Solving the differential inequality (8), one obtains

$$\mu^\alpha(t) V(\xi(t)) \leq \mu^\alpha(0) V(\xi(0)) \exp(-\beta t).$$

Hence, it yields that $V(\xi(t)) \leq \mu^{-\alpha}(t) \exp(-\beta t) V(\xi(0))$. When $t \in [T, \infty)$, due to $\lim_{t\to T^-} \mu^{-\alpha}(t) = 0$ and combination with the continuity of $V(\xi(t))$, we get that $V(\xi(T)) = \lim_{t\to T^-} V(\xi(t)) = 0$. Based on $\dot{V}(\xi(t)) \leq 0$ for $t \geq T$, it has $V(\xi(t)) \equiv 0, t \in [T, \infty)$.

Assumption 1 For any $y_1(t), y_2(t) \in \mathcal{R}^n$, the nonlinear function $f(\cdot, \cdot)$ satisfies

$$\|f(t, y_1(t)) - f(t, y_2(t))\| \leq \rho \|y_1(t) - y_2(t)\|,$$

where ρ is a nonnegative constant.

Assumption 2 The graph $\widetilde{\mathcal{G}}$ contains a directed spanning tree with the leader as the root node, and the subgraph $\mathcal{G}$ is undirected.

Assumption 3 The graph $\widetilde{\mathcal{G}}$ has a directed spanning tree with the leader as the root node, and the subgraph $\mathcal{G}$ is directed.

Lemma 2[17] If Assumption 3 holds, then the matrix $-\mathcal{H}$ is Hurwitz stable. Moreover, there exists a positive diagonal matrix $\mathcal{P} = \mathrm{diag}(\phi_1, \phi_2, \cdots, \phi_N)$ such that $\mathcal{P}\mathcal{H} + \mathcal{H}^T\mathcal{P}$ is positive definite.

2 Main Results

This section analyzes the prescribed-time consensus problem of MASs (1) ~ (2). With the above preparation, the control protocol for agent i is designed as

$$u_i(t) = -\left(k + c\frac{\dot{\mu}(t)}{\mu(t)}\right)\left(\sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)) + b_i(x_i(t) - x_0(t))\right),\ i = 1, 2, \cdots, N \tag{9}$$

where k and c are positive constants.

Define the tracking error as $\tilde{x}_i(t) = x_i(t) - x_0(t)$. The controller (9) can be rewritten in the following compact form

$$u(t) = -\Big(k + c\frac{\dot{\mu}(t)}{\mu(t)}\Big)(\mathcal{H} \otimes I_n)\tilde{x}(t) \tag{10}$$

where $u(t) = [u_1^T(t), u_2^T(t), \cdots, u_N^T(t)]^T$ and $\tilde{x}(t) = [\tilde{x}_1^T(t), \tilde{x}_2^T(t), \cdots, \tilde{x}_N^T(t)]^T$.

Theorem 1 Suppose that Assumptions 1 and 2 hold, then the MASs (1)~(2) can achieve prescribed-time leader-follower consensus under the control protocol (10) within the settling time T, if the positive constants k and c satisfies

$$k > \frac{\rho + \lambda_{\max}(\mathcal{W} + \mathcal{W}^T)/2}{\lambda_{\min}(\mathcal{H})}, \quad c \geq \frac{1}{h\lambda_{\min}(\mathcal{H})} \tag{11}$$

Proof Consider the Lyapunov function as $V(t) = \tilde{x}^T(t)\tilde{x}(t)$. The time derivative of $V(t)$ along the systems (1) ~ (2) are given as

$$\dot{V}(t) = 2\tilde{x}^T(t)\Big[(I_N \otimes \mathcal{W})\tilde{x}(t) - \Big(k + c\frac{\dot{\mu}(t)}{\mu(t)}\Big)(\mathcal{H} \otimes I_n)\tilde{x}(t) + \mathcal{F}(t, x(t)) - 1_N \otimes f(t, x_0(t))\Big] \tag{12}$$

For simplicity, we define $\mathcal{F}(t, x(t)) - 1_N \otimes f(t, x_0(t)) = \tilde{\mathcal{F}}(t, x(t), x_0(t))$, then the above equation can be written as

$$\begin{aligned}\dot{V}(t) =& \tilde{x}^T(t)\Big(I_N \otimes (\mathcal{W} + \mathcal{W}^T)\Big)\tilde{x}(t) - 2\Big(k + c\frac{\dot{\mu}(t)}{\mu(t)}\Big)\tilde{x}^T(t)(\mathcal{H} \otimes I_n)\tilde{x}(t) + 2\tilde{x}^T(t)\tilde{\mathcal{F}}(t, x(t), x_0(t))\\ \leq& -2\Big(k\lambda_{\min}(\mathcal{H}) - \frac{1}{2}\lambda_{\max}(\mathcal{W} + \mathcal{W}^T) - \rho\Big)V(t) - 2\lambda_{\min}(\mathcal{H})c\frac{\dot{\mu}(t)}{\mu(t)}V(t)\end{aligned} \tag{13}$$

According to Lemma 1 and condition (11), it yields

$$V(t) \leq \mu^{-2\lambda_{\min}(\mathcal{H})c}(t)\exp\Big\{-2(k\lambda_{\min}(\mathcal{H}) - \frac{1}{2}\lambda_{\max}(\mathcal{W} + \mathcal{W}^T) - \rho)t\Big\}V(0), \quad t \in [0, T) \tag{14}$$

which further implies that

$$\|\tilde{x}(t)\| \leq \mu^{-\lambda_{\min}(\mathcal{H})c}(t)\exp\Big\{-(k\lambda_{\min}(\mathcal{H}) - \frac{1}{2}\lambda_{\max}(\mathcal{W} + \mathcal{W}^T) - \rho)t\Big\}\|\tilde{x}(0)\|, \quad t \in [0, T) \tag{15}$$

This means that $\lim_{t \to T^-} x_i(t) = x_0(t)$, i.e., the leader-follower consensus is reached within the prescribed-time T.

From the control protocol (10), we can see that $\dot{\mu}(t)/\mu(t)$ will tend to infinity as time approaches T, which will lead the controller (10) to be unbounded. In practical applications, the control protocol $u(t)$ needs to remain bounded. Hence, the boundedness of the controller (10) should be proved over the whole time interval $[0, \infty)$. In light of (15), one gets that

$$\begin{aligned}\|(\mathcal{H} \otimes I_n)\tilde{x}(t)\| \leq& \|\mathcal{H}\|\|\tilde{x}(t)\| \leq \|\mathcal{H}\|\mu^{-c\lambda_{\min}(\mathcal{H})}(t)\exp\Big\{-(k\lambda_{\min}(\mathcal{H}) - \frac{1}{2}\lambda_{\max}(\mathcal{W} + \mathcal{W}^T) - \rho)t\Big\}\|\tilde{x}(0)\|\\ \leq& \|\mathcal{H}\|\|\tilde{x}(0)\|\end{aligned} \tag{16}$$

and

$$\begin{aligned}\|\frac{\dot{\mu}(t)}{\mu(t)}(\mathcal{H} \otimes I_n)\tilde{x}(t)\| =& \frac{h}{T}\mu^{\frac{1}{h}}(t)\|(\mathcal{H} \otimes I_n)\tilde{x}(t)\| \leq \frac{h}{T}\mu^{\frac{1}{h} - c\lambda_{\min}(\mathcal{H})}(t)\exp\Big\{-(k\lambda_{\min}(\mathcal{H}) - \frac{1}{2}\lambda_{\max}(\mathcal{W} + \mathcal{W}^T) - \rho)t\Big\}\|\mathcal{H}\|\|\tilde{x}(0)\|\\ \leq& \frac{h}{T}\|\mathcal{H}\|\|\tilde{x}(0)\|\end{aligned} \tag{17}$$

Combining (16) and (17), we have

$$\|u(t)\| \leq k\|(\mathcal{H} \otimes I_n)\tilde{x}(t)\| + c\|\frac{\dot{\mu}(t)}{\mu(t)}(\mathcal{H} \otimes I_n)\tilde{x}(t)\| \leq (k + c\frac{h}{T})\|\mathcal{H}\|\|\tilde{x}(0)\|, \quad t \in [0, T) \tag{18}$$

Next, we will discuss that the consensus is kept and the control protocol $u(t) \equiv 0$ over $[T, \infty)$. Choose the same Lyapunov function candidates as in Theorem 1, and following the same procedure from (12) to (13), it can be learned that

$$\dot{V}(t) \leq -2\Big(k\lambda_{\min}(\mathcal{H}) - \frac{1}{2}\lambda_{\max}(\mathcal{W} + \mathcal{W}^T) - \rho\Big)V(t) \leq 0, \quad t \in [T, \infty) \tag{19}$$

Note that $V(t)$ is continuous at $t=T$, then it follows

$$V(T)=\lim_{t\to T^-}\tilde{x}^T(t)\tilde{x}(t)=0 \tag{20}$$

Combining (19) with (20), it yields

$$0\le V(t)\le V(T)=0,\quad t\in[T,\infty) \tag{21}$$

We can easily get $V(t)\equiv 0$, $t\ge T$. Furthermore, we obtain $u(t)\equiv 0$ on $[T,\infty)$, which imply

$$\begin{cases} \|u(t)\|\le (k+c\frac{h}{T})\|\mathcal{H}\|\|\tilde{x}(0)\|, & t\in[0,T) \\ \|u(t)\|=0, & t\in[T,\infty) \end{cases} \tag{22}$$

In other words, the prescribed-time leader-follower consensus of the systems (1)~(2) are proved, and the control law $u(t)$ is always bounded for $t\in[0,\infty)$.

Remark 1 It should be pointed out that the control law (10) is significant for achieving the prescribed-time consensus. However, the function $\mu^{1/h}(t)\to\infty$ as $t\to T$. We need to demonstrate that the control law (10) is bounded. According to the preceding analysis, the controller $u(t)$ is always bounded over the whole time interval $[0,\infty)$, so our design is reasonable.

Due to the fact that information sharing between MASs is not always bidirectional, studying the consensus of MASs on directed network topology is more meaningful. Therefore, the above results are extended to general directed network topology.

Theorem 2 If Assumptions 1 and 3 hold and there exist positive constants k and c such that

$$k>\frac{\lambda_{\max}(\mathcal{P}\mathcal{H}\mathcal{H}^T\mathcal{P})}{\tilde{\lambda}}+\frac{2(\|\mathcal{W}\|^2+\rho^2)}{\lambda_{\min}(\mathcal{H}\mathcal{H}^T)\tilde{\lambda}},\quad c\ge\frac{2\phi_{\max}}{\tilde{\lambda}h} \tag{23}$$

then the MASs (1) ~ (2) with the control protocol (10) can achieve the prescribed-time consensus.

Proof Construct Lyapunov function

$$V(t)=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n}\phi_i\hbar_{ij}^2(t) \tag{24}$$

where $\hbar_i(t)=\sum_{j=1}^N a_{ij}(x_i(t)-x_j(t))+b_i(x_i(t)-x_0(t))$, $\hbar(t)=[\hbar_1^T(t),\hbar_2^T(t),\cdots,\hbar_N^T(t)]^T$ with $\hbar_i(t)=[\hbar_{i1}(t),\hbar_{i2}(t),\cdots,\hbar_{in}(t)]^T$.

Taking the time derivative of the Lyapunov function (24), it yields

$$\begin{aligned}
\dot{V}(t)&=\sum_{i=1}^{N}\sum_{j=1}^{n}\phi_i\hbar_{ij}(t)\dot{\hbar}_{ij}(t)=\hbar^T(t)(\mathcal{P}\otimes I_n)\dot{\hbar}(t)\\
&=-\Big(k+c\frac{\dot{\mu}(t)}{\mu(t)}\Big)\hbar^T(t)(\mathcal{P}\mathcal{H}\otimes I_n)\hbar(t)\\
&\quad+\hbar^T(t)(\mathcal{P}\mathcal{H}\otimes I_n)\Big[(I_n\otimes\mathcal{W})\tilde{x}(t)+\mathcal{F}(t,x(t))-1_N\otimes f(t,x_0(t))\Big]\\
&\le-\frac{1}{2}\Big(k+c\frac{\dot{\mu}(t)}{\mu(t)}\Big)\hbar^T(t)\Big((\mathcal{P}\mathcal{H}+\mathcal{H}^T\mathcal{P})\otimes I_n\Big)\hbar(t)\\
&\quad+\frac{1}{2}\Big(\Big\|(\mathcal{H}^T\mathcal{P}\otimes I_n)\hbar(t)\Big\|^2+\Big\|(I_n\otimes\mathcal{W})\tilde{x}(t)+\mathcal{F}(t,x(t))-1_N\otimes f(t,x_0(t))\Big\|^2\Big)\\
&\le-\frac{1}{2}\Big(k+c\frac{\dot{\mu}(t)}{\mu(t)}\Big)\tilde{\lambda}\hbar^T(t)\hbar(t)+\frac{1}{2}\|(\mathcal{H}^T\mathcal{P}\otimes I_n)\hbar(t)\|^2+\|(I_n\otimes\mathcal{W})\tilde{x}(t)\|^2\\
&\quad+\|\mathcal{F}(t,x(t))-1_N\otimes f(t,x_0(t))\|^2
\end{aligned} \tag{25}$$

According to Lemma 2, we get $\tilde{\lambda}=\lambda_{\min}(\mathcal{P}\mathcal{H}+\mathcal{H}^T\mathcal{P})>0$. By utilizing Assumption 1, it has

$$\|\mathcal{F}(t,x(t))-1_N\otimes f(t,x_0(t))\|^2=\sum_{i=1}^{N}\|f(t,x_i(t))-f(t,x_0(t))\|^2\le\frac{\rho^2}{\lambda_{\min}(\mathcal{H}\mathcal{H}^T)}\|\hbar(t)\|^2 \tag{26}$$

Substituting (26) into (25), one can get that

$$\dot{V}(t) \le -\frac{1}{2}\Big(k + c\frac{\dot{\mu}(t)}{\mu(t)}\Big)\tilde{\lambda}\hbar^T(t)\hbar(t) + \frac{1}{2}\lambda_{\max}(\mathcal{PHH}^T\mathcal{P})\|\hbar(t)\|^2 + \frac{\|\mathcal{W}\|^2+\rho^2}{\lambda_{\min}(\mathcal{HH}^T)}\|\hbar(t)\|^2$$
$$= -\Big(\frac{\tilde{\lambda}k}{2} - \frac{1}{2}\lambda_{\max}(\mathcal{PHH}^T\mathcal{P}) - \frac{\|\mathcal{W}\|^2+\rho^2}{\lambda_{\min}(\mathcal{HH}^T)}\Big)\sum_{i=1}^{N}\sum_{j=1}^{n}\hbar_{ij}^2(t) - \frac{\tilde{\lambda}c}{2}\frac{\dot{\mu}(t)}{\mu(t)}\sum_{i=1}^{N}\sum_{j=1}^{n}\hbar_{ij}^2(t) \tag{27}$$

From the definition of $V(t)$, one has

$$\dot{V}(t) \le -\frac{2}{\phi_{\max}}\Big(\frac{\tilde{\lambda}k}{2} - \frac{1}{2}\lambda_{\max}(\mathcal{PHH}^T\mathcal{P}) - \frac{\|\mathcal{W}\|^2+\rho^2}{\lambda_{\min}(\mathcal{HH}^T)}\Big)V(t) - \frac{\tilde{\lambda}c}{\phi_{\max}}\frac{\dot{\mu}(t)}{\mu(t)}V(t) \tag{28}$$

where $\phi_{\max} = \max\{\phi_1, \phi_2, \cdots, \phi_N\}$. By using condition (23), one can obtain

$$\dot{V}(t) \le -\alpha\frac{\dot{\mu}(t)}{\mu(t)}V(t) - \beta V(t) \tag{29}$$

where $\alpha = \tilde{\lambda}c/\phi_{\max}$, $\beta = (2/\phi_{\max})\big((\tilde{\lambda}k/2) - (1/2)\lambda_{\max}(\mathcal{PHH}^T\mathcal{P}) - [(\|\mathcal{W}\|^2+\rho^2)/\lambda_{\min}(\mathcal{HH}^T)]\big)$.

According to Lemma 1, we obtain $V(t) \le \mu^{-\alpha}(t)\exp(-\beta t)V(0), t \in [0, T)$, and $V(t) \equiv 0, t \in [T, \infty)$. This means that $\lim_{t\to T^-} x_i(t) = x_0(t)$, and $x_i(t) = x_0(t)$ for $t \ge T$, i.e., the prescribed-time consensus problem is solved under the directed network topology.

Taking a similar process as Theorem 1, we can derive that $u(t)$ is bounded over the whole time interval $[0, \infty)$.

Remark 2 It should be pointed out that the settling time of finite-time algorithms is related to the initial state, i.e., a large initial state will increase the settling time. In addition, for fixed-time consensus protocol, the settling time is determined by the parameters of the system. However, in our proposed algorithms, the settling time T is not only regardless with the initial values of the system, but also does not depend on control parameters.

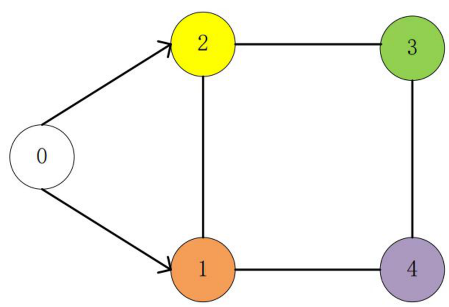

Fig 1 Communication topology

3 Numerical Simulations

In this section, two examples are presented to verify the effectiveness of the obtained results.

Example 1 Consider MASs (1) ~ (2) with one leader and four followers, in which $\mathcal{W} = \begin{pmatrix} 0 & 0.5 \\ -2 & 0 \end{pmatrix}$, the nonlinear function is $f(t, x_i(t)) = [\sin(x_{i1}(t)), \cos(x_{i2}(t))]^T, i = 0, 1, 2, 3, 4$, and $\rho = 1$. The communication graph is given as in Fig 1. The corresponding Laplacian matrix among the followers is $\mathcal{L} = \begin{pmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{pmatrix}$ and $\mathcal{B} = \mathrm{diag}(1,1,0,0)$. Choosing the initial states $[x_{01}(0), x_{11}(0), x_{21}(0), x_{31}(0), x_{41}(0)] = [-3, -4.3, 2, -7, 3]$, $[x_{02}(0), x_{12}(0), x_{22}(0), x_{32}(0), x_{42}(0)] = [5.5, 7.3, 2, 5, 11]$. According to Theorem 1, choose the other parameters $c = 3$, $k = 6.54$ and $T = 0.5$. Figs 2~3 depict the state trajectories of $x_i(t)$, $i = 0, \cdots, 4$. Figs 4~5 depict the state error of $\tilde{x}_i(t)$, $i = 1, \cdots, 4$. It can be seen that the system achieves prescribed-time leader-follower consensus within $T = 0.5$.

Example 2 We consider MASs (1) ~ (2) consist of five agents, which one leader agent and the remainder as followers. The communication graph is given in Fig 6. The nonlinear function is $f(t, x_i(t)) = [\sin(x_{i1}(t)), \cos(x_{i2}(t))]^T, i = 0, 1, 2, 3, 4$. It can be seen that $f(t, x_i(t))$ satisfies Assumption 1 with $\rho = 1$. Based on Lemma 2, we have $\mathcal{P} = \mathrm{diag}(2,3,1,2)$. The system matrices are given as $\mathcal{W} = \begin{pmatrix} 1 & -3 \\ 1 & 0 \end{pmatrix}$. Meanwhile, the initial conditions of the systems (1) ~ (2) are given as $[x_{01}(0), x_{11}(0), x_{21}(0), x_{31}(0), x_{41}(0)] = [-2, -5.3, 1, 3, -4]$, $[x_{02}(0), x_{12}(0), x_{22}(0), x_{32}(0), x_{42}(0)] = [1, 2, 3, 1.5, -1]$. In addition, set $c = 3.66$, $k = 28.861$, $T = 0.5$. Figs 7~8 show the state trajectories of $x_i(t)$, $i = 0, \cdots, 4$. Figs 9~10 show the state error of $\tilde{x}_i(t)$, $i = 1, \cdots, 4$. Obviously, the prescribed-time control protocol ensures that the MASs achieves leader-follower consensus within $T = 0.5$.

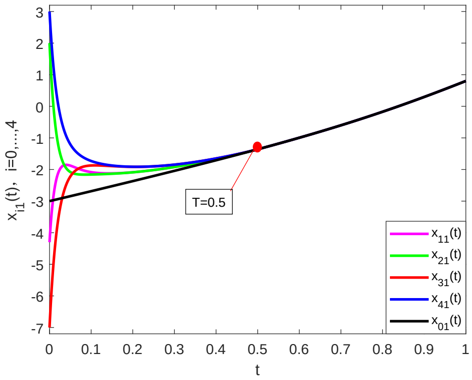

Fig 2 The state trajectories of x_{i1}

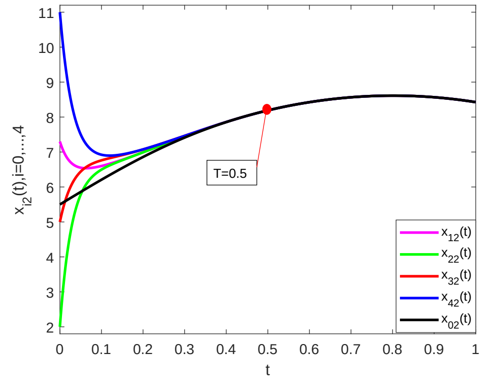

Fig 3 The state trajectories of x_{i2}

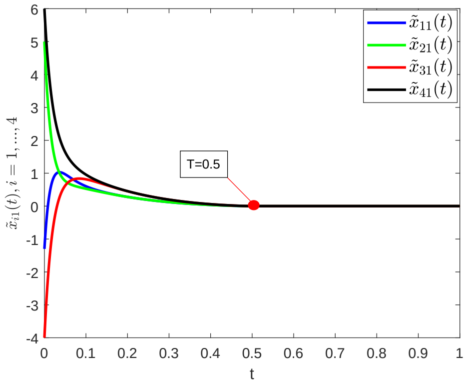

Fig 4 The state error of $\tilde{x}_{i1}$

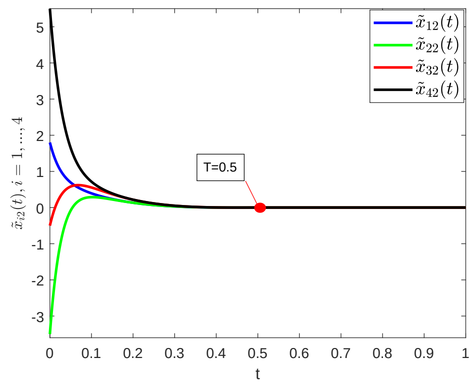

Fig 5 The state error of $\tilde{x}_{i2}$

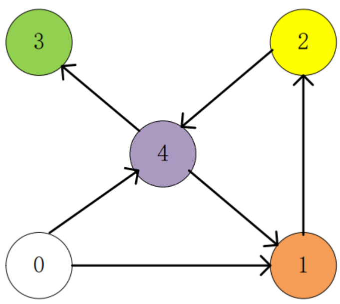

Fig 6 Communication topology

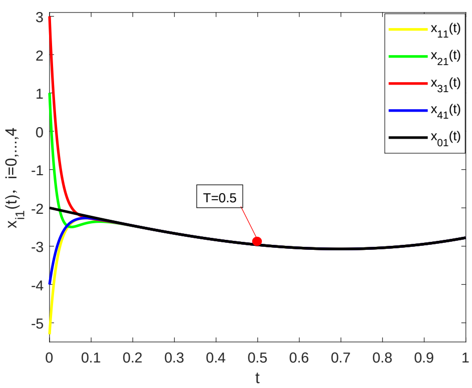

Fig 7 The state trajectories of x_{i1}

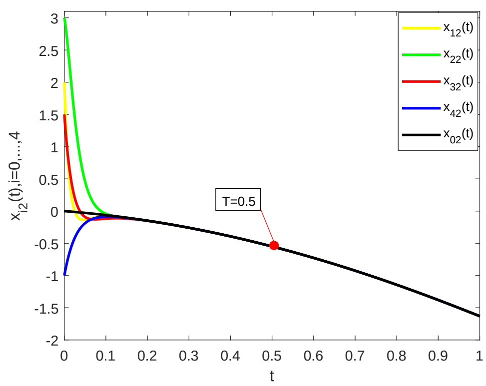

Fig 8 The state trajectories of x_{i2}

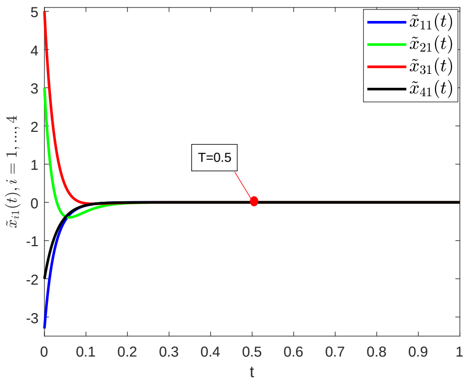

Fig 9 The state error of $\tilde{x}_{i1}$

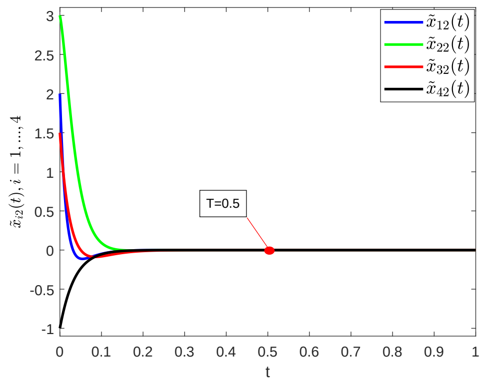

Fig 10 The state error of $\tilde{x}_{i2}$

4 Conclusion

In this paper, a new control scheme was constructed to achieve prescribed-time leader-follower consensus for first-order nonlinear MASs under undirected and directed network topologies. Based on Lyapunov stability theory, some sufficient conditions are derived to achieve leader-follower consensus. The proposed control scheme is bounded over the whole time interval. The prescribed-time consensus algorithm can ensure that the settling time is arbitrarily preallocated and is independent of the initial state and other parameters. This paper mainly focuses on only one leader prescribed-time consensus problem of first-order MASs. In future work, we will focus our research topic on prescribed-time consensus problems with second-order and high-order dynamics.

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