Selection of Constitutive Models for As-Cast Low Alloyed Al-*x*Si-*y*Cu Alloys*

GAO Xiang

(*School of Materials Science and Engineering, Xinjiang University, Urumqi Xinjiang 830017, China*)

Abstract: To guarantee the computational accuracy of the finite element model, the strain-compensated Arrhenius-type model, modified Fields-Backofen (m-FB) model and modified Zerilli-Armstrong (m-ZA) model were established to predict the high-temperature flow stress of as-cast low alloyed Al-0.5Cu, Al-1Si, and Al-1Si-0.5Cu. To determine the material constants of these three constitutive models, isothermal compression tests of the three aluminum alloys were carried out on a Gleeble-3800 thermal simulator. The prediction results of the constitutive model were compared with the experimental results to evaluate the prediction accuracy of the constitutive models, and to provide a basis for selecting the most suitable constitutive models (parameters) for the three alloys mentioned above. It is found that the strain-compensated Arrhenius model and m-ZA model can be regarded as the most suitable constitutive models for Al-0.5Cu and Al-1Si alloys, respectively, and these two constitutive models also can be applied to Al-1Si-0.5Cu alloy. However, the m-FB model can be applied to Al-0.5Cu, Al-1Si and Al-1Si-0.5Cu alloys only under high temperature and medium strain conditions.

Key words: constitutive model; flow stress; hot deformation; aluminum alloy

DOI：10.13568/j.cnki.651094.651316.2024.01.16.0001

CLC number：TH142.2　**Document Code**：A　**Article ID**：2096-7675(2025)01-0001-013

引文格式：高翔．铸态低合金Al-*x*Si-*y*Cu本构模型的选择[J]．新疆大学学报(自然科学版中英文)，2025，42(1)：1-13+23．

英文引文格式：GAO Xiang．Selection of constitutive models for as-cast low alloyed Al-*x*Si-*y*Cu alloys[J]．Journal of Xinjiang University(Natural Science Edition in Chinese and English)，2025，42(1)：1-13+23．

铸态低合金Al-*x*Si-*y*Cu本构模型的选择

高 翔

(新疆大学 材料科学与工程学院，新疆 乌鲁木齐 830017)

摘　要： 为保证有限元模型的计算精度，建立了应变补偿Arrhenius模型、修正的Fields-Backofen（m-FB）模型和修正的Zerilli-Armstrong（m-ZA）模型来预测铸态低合金Al-0.5Cu、Al-1Si和Al-1Si-0.5Cu的高温流变应力．为了确定三种本构模型的材料常数，在Gleeble-3800热模拟机上对三种铝合金进行了等温压缩试验．本构模型的预测结果与试验结果作比较以评估本构模型预测的准确性，并为上述三种合金选择最合适的本构模型（参数）提供依据．研究表明，应变补偿Arrhenius模型和m-ZA模型可分别作为Al-0.5Cu和Al-1Si合金最合适的本构模型，这两种本构模型同样可应用于Al-1Si-0.5Cu合金．然而，m-FB模型只能在高温和中等应变条件下应用于Al-0.5Cu、Al-1Si和Al-1Si-0.5Cu合金．

关键词： 本构模型；流变应力；热变形；铝合金

0 Introduction

Finite element method (FEM) is an effective tool for casting process design and optimization, which can assist in analyzing the temperature field, flow field, and stress-strain distribution during casting under different conditions. However, the problem most frequently encountered in the process of establishing finite element model is lack of required input parameters. In a sense, the reliability of finite element model highly depends on the accuracy of input parameters. Constitutive parameters are one of the essential input parameters in the thermal-mechanical finite element model, mainly determined through the

* **Received Date:** 2024-01-16

Foundation Item: This work was supported by the Natural Science Foundation of Xinjiang Uygur Autonomous Region of China "Study on constitutive behavior of Al-*x*Si-*y*Cu high purity aluminum alloy billets for target materials"（2020D01C023）.

Biography: GAO Xiang（1980—），male，doctor，associate professor，research field：metallic materials，E-mail：gaoxiang@xju.edu.cn.

constitutive model. Generally speaking, constitutive models can be divided into three categories: phenomenological model, physical-based model and artificial neural network model[1].

The phenomenological constitutive model is based on experimental observations, with fewer material parameters and easy calibration. The model is frequently used to simulate the hot deformation under high strain rates and temperatures. However, phenomenological models are sometimes unable to accurately describe the microscopic phenomena in the forming process of metals or alloys due to lack of physical background consistent with experimental observations.

Arrhenius-type constitutive model[2] is one of the phenomenological models, which is widely used to simulate the hot deformation behavior and stress-strain distribution of aluminum alloys[3−6]. For instance, Rudnytskyj et al.[3] studied the hot deformation behavior of 6061 aluminum alloy under a wide range of temperatures and strain rates by Arrhenius-type, Johnson-Cook (JC) and Hensel-Spittel constitutive models. The results show that the Arrhenius-type constitutive model has the highest accuracy compared with the other two models by coupling the strain into the model with high-order polynomials. Chamanfar et al.[4] established a power-law Arrhenius constitutive model to predict the flow stress of AA6099 alloy under given experimental conditions. It is found that the dynamic recrystallization (DRX) fraction increases and the average size of grain and sub-grain decreases with the increase of Zener-Hollomon parameter. Wu et al.[5] conducted isothermal compression test of 7050 aluminum alloy and studied flow stress features during high temperature plastic deformation by Arrhenius-type model. By analyzing the relationship between microstructure and material parameters, it is found that DRX volume fraction increases with the increase of strain and deformation temperature, and decreases with the increase of strain rate. Shi et al.[6] found that different constitutive parameters should be used to describe the hot compression deformation behavior of 6005A aluminum alloy at different temperature ranges. The reason is that the deformation mechanisms involved are different at different deformation conditions. Therefore, two groups of different exponential Arrhenius models are proposed in the temperature ranges of 573~623 K and 623~773 K.

Fields-Backofen (FB) constitutive model is another commonly used phenomenological model, in which strain hardening coefficient (n) and strain rate sensitivity index (m) are used. A larger number of researchers have studied the constitutive behavior of metallic materials by FB model[7−11]. For example, Tsao et al.[9] used FB model to describe the flow behavior of industrial pure titanium sheet during warm tensile deformation. The results show that the predicted results in strain hardening and softening stages are in good agreement with the experimental results. Based on isothermal compression tests of 21-4N heat-resistant steel, Huang et al.[10] established FB, modified-FB (m-FB) and modified Zerilli-Armstrong (m-ZA) models to predict the flow stress of 21-4N steel. It is found that the original FB model has high prediction ability only at low temperature and low strain rate. Both m-FB and m-ZA models can well describe the flow behavior of 21-4N heat resistant steel under various deformation conditions. Jia et al.[11] studied the constitutive behavior of AZ31B magnesium alloy during hot compression by m-FB model. It is found that the modified model is applicable not only to as-cast AZ31B magnesium alloy, but also to other metal materials with similar plastic deformation mechanism.

Compared with the phenomenological model, the physical-based model, basically from a microcosmic perspective, considers the dislocation dynamics, thermal activation and some other mechanisms in the process of plastic deformation. Through some physical assumptions and a large number of material constants, the physical-based constitutive model can predict the flow stress in a wide range of deformation conditions accurately, especially at high temperature and strain rate.

Zerilli-Armstrong model[12] is a representative physical-based constitutive model, which has gained extensive application in the prediction of flow stress of various materials[13−19]. Paturi et al.[19] established the modified Johnson-Cook (m-JC) model and the m-ZA model to predict the flow stress of high strength aluminum alloy AA7075-T6 under a wide range of temperatures and strain rates. The research shows that both m-JC model and m-ZA model have good prediction results. However, the mean absolute error of the m-ZA model is lower than that of the m-JC model, which means that the m-ZA model can provide more reliable prediction results for the flow stress of AA7075-T6 aluminum alloy. Zhan et al.[20] studied the flow behavior of β-titanium alloy Ti6554 by JC and m-ZA models, and found that m-ZA model can better describe the strain hardening behavior of Ti6554 alloy as it incorporates the coupling effects of strain and temperature.

Al-0.5Cu, Al-1Si, and Al-1Si-0.5Cu are three low alloyed aluminum alloys commonly used for sputtering targets. One of the main research contents of current study is to establish thermal-mechanical finite element models of these three alloys during DC casting process. In this work, to obtain the input parameters required for the finite element model, strain-compensated Arrhenius-type, m-FB and m-ZA constitutive models were established. The material constants of these constitutive models are determined by the true stress-strain curves obtained from isothermal compression tests. The high-temperature flow stress of Al-0.5Cu, Al-1Si, and Al-1Si-0.5Cu as-cast aluminum alloys predicted by the above three constitutive models was compared with experimental testing results. By comparing and analyzing the differences in prediction accuracy caused by changes in constitutive model and alloy system, a basis is provided for selecting the most suitable constitutive model (parameters), thereby ensuring the reliability of the results obtained from the FE model.

1 Experimental

1.1 DSC Measurements

The Al-0.5Cu, Al-1Si and Al-1Si-0.5Cu alloys used in the study were produced by DC casting in an industrial production line. The purity of aluminum and copper used in the preparation of alloys is above 99.999 0% (5N), and the purity of silicon is above 99.999 9% (6N). To clarify the differences in alloy properties caused by the addition of Si and Cu, DSC measurements of as-cast specimens were carried out. For comparison, high-purity aluminum specimen with a purity of 99.999 5% (5N5) was also measured. The mass of each DSC specimen was about 30 mg. The specimens for DSC measurement were equilibrated at 298 K (25 ℃) and then heated to 1 023 K (750 ℃) with a heating rate of 10 K/min under an argon atmosphere. The measurements were performed two times for each specimen to ensure good reproducibility.

1.2 Hot Compression Tests

Cylindrical samples with a diameter of 10 mm and a height of 15 mm are prepared by wire-electrode cutting to meet the requirements of the following mechanical tests. The high temperature flow behavior of the three aluminum alloys was investigated by isothermal compression tests with a Gleeble-3800 thermal simulator. Experimental procedure for hot compression tests is shown in Fig 1. During the test, the specimen is heated from room temperature to the preset deformation temperature at the heating rate of 20 K/s and held for 5 minutes to eliminate the temperature gradient. After that, the compression tests were performed under temperatures of 473 K, 573 K, 623 K, 673 K, 723 K and 773 K with strain rates of 10^{-3} s^{-1}, 10^{-2} s^{-1}, 10^{-1} s^{-1} and 1 s^{-1}. All specimens were quenched by water after deformation to retain the microstructures.

2 Results and Discussion

2.1 DSC Curves

Fig 2 shows the results of DSC measurements. It can be seen that the four curves in Fig 2 are very similar. Except for the melting peak, no other peaks were found, indicating that the three studied low alloyed alloys were all solid solution strengthened aluminum alloys without the formation of a second phase.

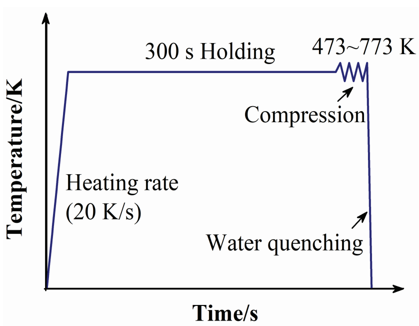

Fig 1 Experimental procedure for hot compression tests

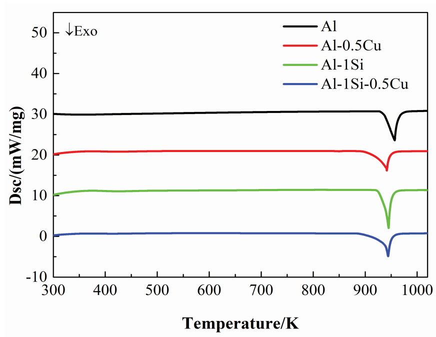

Fig 2 DSC curves of 5N5 high-purity Al and Al-*x*Si-*y*Cu alloys

2.2 True Stress-Strain Curves

The true stress-strain curves of Al-0.5Cu, Al-1Si and Al-1Si-0.5Cu alloys obtained from the tests are shown in Figs 3~5, respectively. From these figures, it can be seen that deformation temperatures and strain rates have a significant effect on flow stress. The flow stress decreases with the increase of temperatures and increases with the increase of strain rates. At the early stage of deformation, the dislocation density rises rapidly, and work hardening is dominant, resulting in the rapid increase of flow stress. With the occurrence of dynamic softening, the effect of work hardening is gradually weakened, which slows down the increase of flow stress. Thus, the flow stress reaches the peak stress and remains basically constant. In some cases, a decrease in flow stress is observed because the effect of dynamic softening is greater than that of work hardening. Generally speaking, work hardening is dominant at lower temperature and higher strain rate. Correspondingly, dynamic softening is dominant at higher temperature and lower strain rate. This is because higher temperature and lower strain rate provide higher grain boundary mobility and more sufficient time for nucleation and growth of dynamically recrystallized grains[21]. Comparing Figs 3~5, it can be seen that under the same deformation conditions, the peak stress of the three alloys from high to low are Al-1Si-0.5Cu, Al-0.5Cu and Al-1Si, reflecting that the strength of aluminum alloys is affected not only by the content of alloy elements, but also by the types of alloy elements.

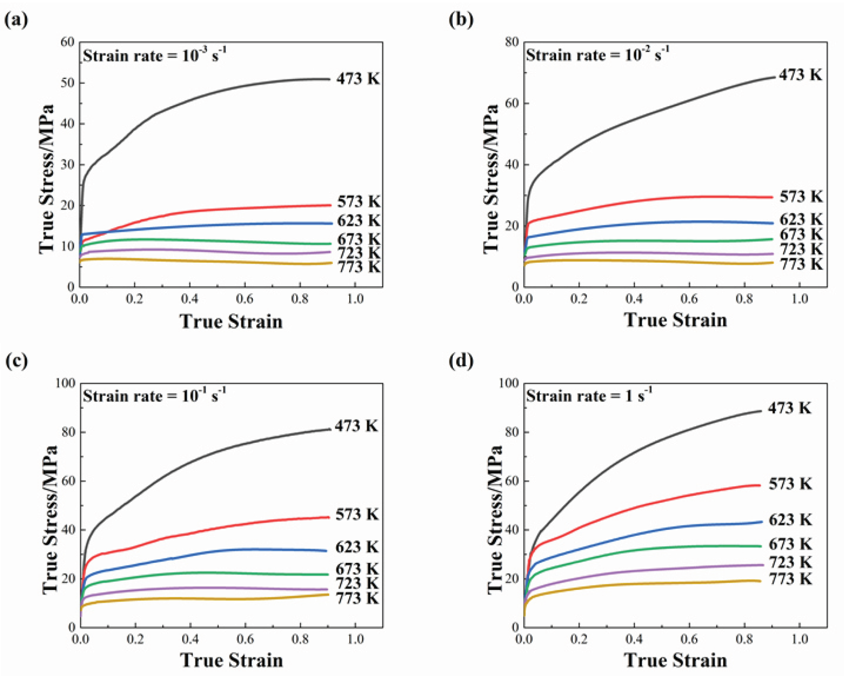

Fig 3 True stress-strain curves for Al-0.5Cu alloy under different deformation temperatures with strain rates of (a) 10^{-3} s^{-1}; (b) 10^{-2} s^{-1}; (c) 10^{-1} s^{-1} and (d) 1 s^{-1}

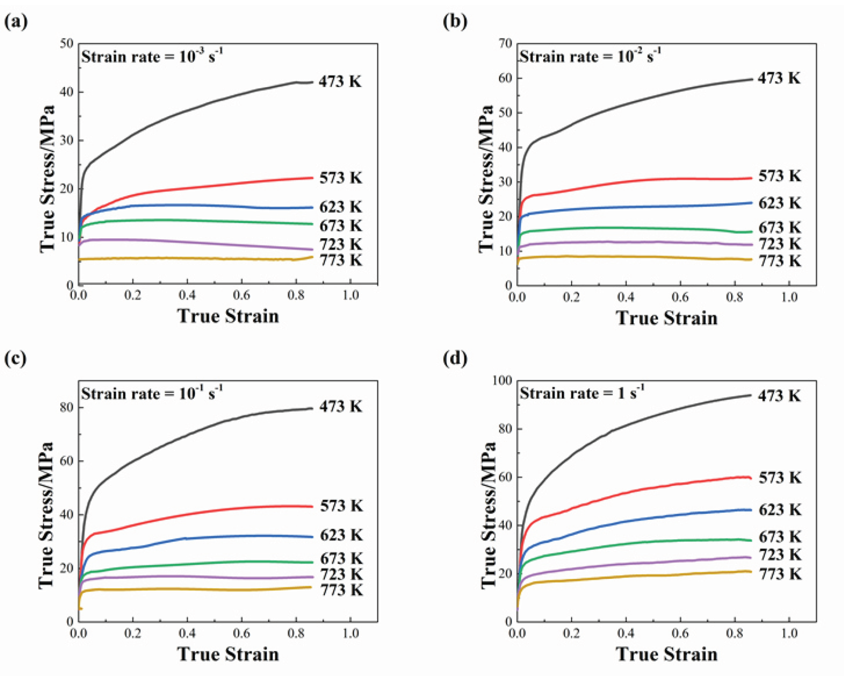

Fig 4 True stress-strain curves for Al-1Si alloy under different deformation temperatures with strain rates of (a) 10^{-3} s^{-1}; (b) 10^{-2} s^{-1}; (c) 10^{-1} s^{-1} and (d) 1 s^{-1}

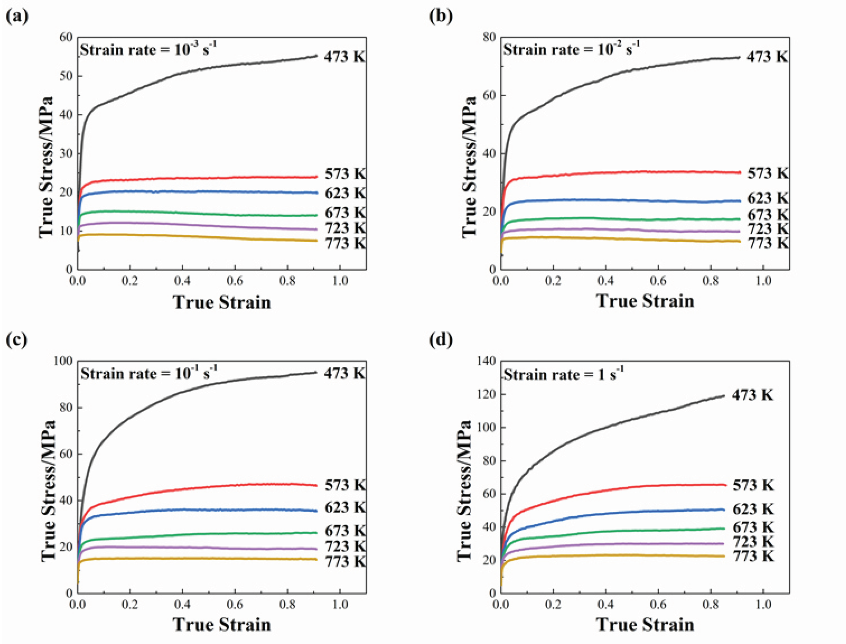

Fig 5 True stress-strain curves for Al-1Si-0.5Cu alloy under different deformation temperatures with strain rates of (a) 10^{-3} s^{-1}; (b) 10^{-2} s^{-1}; (c) 10^{-1} s^{-1} and (d) 1 s^{-1}

2.3 Determination of Constitutive Parameters

The true stress-strain data obtained from isothermal compression tests were employed to determine the material constants of the constitutive models. In current studies, the material constants were calculated under different deformation strains within the range of 0.02 to 0.84 at the interval of 0.02.

2.3.1 Strain-Compensated Arrhenius-Type Model

The general expression of Arrhenius-type constitutive model includes the following equations:

$$Z = \dot{\varepsilon} \exp \frac{Q}{RT} \tag{1}$$

$$\dot{\varepsilon} = A f(\sigma) \exp(-\frac{Q}{RT}) \tag{2}$$

$$f(\sigma) = \begin{cases} \sigma^{n_1}, & \alpha\sigma < 0.8 \\ \exp(\beta\sigma), & \alpha\sigma > 1.2 \\ [\sinh(\alpha\sigma)]^n, & \text{for all } \sigma \end{cases} \tag{3}$$

where Z is Zener-Hollomon parameter, $\dot{\varepsilon}$ is strain rate (s^{-1}), Q is the activation energy (kJ · mol^{-1}), R is gas constant (8.314 J · mol^{-1}· K^{-1}), T is the absolute temperature (K), σ is true stress (MPa). α, β and n are temperature independent material constants and $\alpha = \beta / n_1$.

Fig 6 shows the methods and steps for determining the material constants in Arrhenius-type constitutive model. After the material constant is determined, the flow stress can be calculated using the Arrhenius-type constitutive model, but no strain is specified when describing the flow stress. Therefore, strain compensation should be considered to improve the prediction ability of the model. Polynomial fitting can be used for strain compensation to establish the relationship between the material constants (α, n, Q and $\ln A$) and the true strain. In current study, it is found that the sixth-order polynomial fitting has the best correlation and generalization. Polynomials of higher order (i.e.>6) are overfitted, thus losing their generalization ability. The polynomial fitting expression is shown in equation (4), and the fitting results of α, n, Q and $\ln A$ are provided in Table 1.

$$\begin{cases} \alpha = C_0 + C_1\varepsilon + C_2\varepsilon^2 + C_3\varepsilon^3 + C_4\varepsilon^4 + C_5\varepsilon^5 + C_6\varepsilon^6 \\ n = D_0 + D_1\varepsilon + D_2\varepsilon^2 + D_3\varepsilon^3 + D_4\varepsilon^4 + D_5\varepsilon^5 + D_6\varepsilon^6 \\ Q = E_0 + E_1\varepsilon + E_2\varepsilon^2 + E_3\varepsilon^3 + E_4\varepsilon^4 + E_5\varepsilon^5 + E_6\varepsilon^6 \\ \ln A = F_0 + F_1\varepsilon + F_2\varepsilon^2 + F_3\varepsilon^3 + F_4\varepsilon^4 + F_5\varepsilon^5 + F_6\varepsilon^6 \end{cases} \tag{4}$$

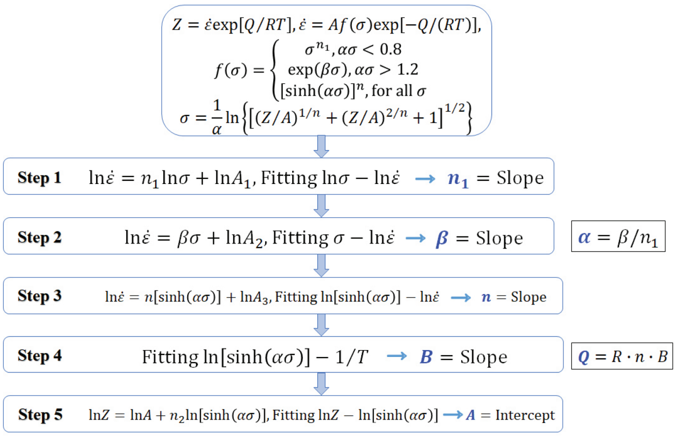

Fig 6 Methods and steps for determining material constants of Arrhenius-type constitutive model

Table 1 Polynomial fitting coefficient of α, n, Q and $\ln A$

Alloys	α	n	Q	$\ln A$
Al-0.5Cu	C_0=0.071 4	D_0=6.120 4	E_0=91.270 1	F_0=11.090 7
	C_1=-0.227 4	D_1=22.298 5	E_1=942.08	F_1=198.146 8
	C_2=0.856 9	D_2=-181.429 6	E_2=-5 633.54	F_2=-1 155.750 0
	C_3=-1.896 6	D_3=548.387 3	E_3=16 135.10	F_3=3 263.247 8
	C_4=2.477 7	D_4=-830.203 3	E_4=-24 095.80	F_4=-4 834.315 8
	C_5=-1.773 6	D_5=626.480 4	E_5=18 114.30	F_5=3 622.848 0
	C_6=0.536 2	D_6=-187.429 5	E_6=-5 420.61	F_6=-1 085.207 4
Al-1Si	C_0=0.053 9	D_0=7.402 5	E_0=99.020 3	F_0=13.544 7
	C_1=-0.006 2	D_1=5.267 4	E_1=68.220 1	F_1=146.440 0
	C_2=-0.233 4	D_2=-92.355 7	E_2=-3 441.034 1	F_2=-784.983 4
	C_3=0.916 2	D_3=305.523 5	E_3=8 462.350 2	F_3=2 052.926 8
	C_4=-1.497 7	D_4=-468.963 6	E_4=-10 881.502 3	F_4=-2 826.699 2
	C_5=1.126 3	D_5=350.184 8	E_5=6 969.810 1	F_5=1 969.246 8
	C_6=-0.317 3	D_6=-102.518 9	E_6=-1 741.130 2	F_6=-548.131 4
Al-1Si-0.5Cu	C_0=0.042 7	D_0=7.487 5	E_0=124.290 2	F_0=17.054 2
	C_1=-0.049 4	D_1=-13.023 5	E_1=232.740 1	F_1=71.266 7
	C_2=0.170 1	D_2=28.322 2	E_2=-1 278.090 3	F_2=-393.180 4
	C_3=-0.395 2	D_3=-35.805 7	E_3=3 489.180 1	F_3=1 089.047 1
	C_4=0.606 3	D_4=32.548 5	E_4=-4 967.460 2	F_4=-1 592.769 5
	C_5=-0.514 8	D_5=-27.082 2	E_5=3 572.470 4	F_5=1 183.413 8
	C_6=0.178 1	D_6=12.378 6	E_6=-1 027.480 1	F_6=-352.568 5

Once the material constant is determined, the flow stress (σ) can be written as a function of the Zener-Hollomon parameter (Z), and the equation can be mathematically expressed as:

$$\sigma = \frac{1}{\alpha}\ln\left\{\left[\left(\frac{Z}{A}\right)^{\frac{1}{n}} + \left(\frac{Z}{A}\right)^{\frac{2}{n}} + 1\right]^{\frac{1}{2}}\right\} \tag{5}$$

where α, n, Q and A could be obtained from (4).

2.3.2 Modified Fields-Backofen (m-FB) Model

The original FB model has a simple structure and its expression is as follows:

$$\sigma = K\varepsilon^n\dot{\varepsilon}^m \tag{6}$$

where K is the strength coefficient, n is the strain hardening exponent, m is the strain-rate sensitivity exponent.

The original FB model is in exponential form, which means that its prediction results show an exponential upward trend and no descending stage. Obviously, it cannot correspond to the reality of true stress-strain curves. To overcome the shortcomings of the original FB model, the model can be modified by introducing a softening term[10]. The expression is as follows:

$$\sigma = K_1 \varepsilon^n \dot{\varepsilon}^m \exp(bT + s\varepsilon) \tag{7}$$

where b is thermal effect constant, s is work softening impact index.

The methods and steps for determining the material constants in m-FB model are shown in Fig 7. Table 2 shows the constitutive equations of Al-0.5Cu, Al-1Si and Al-1Si-0.5Cu aluminum alloys obtained by the m-FB model.

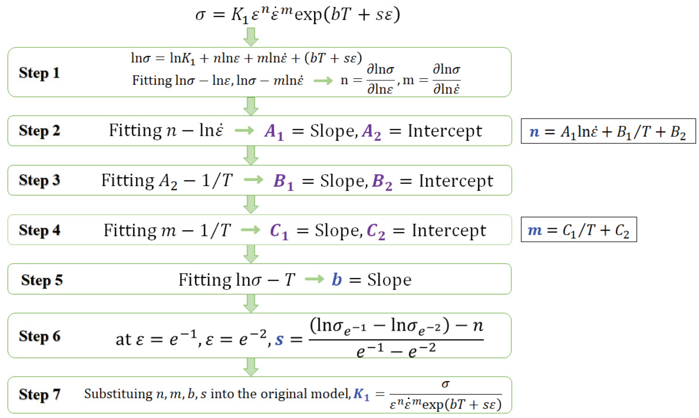

Fig 7 Methods and steps for determining material constants of m-FB model

Table 2 Constitutive equations determined by the m-FB model

Alloys	Constitutive equations
Al-0.5Cu	$\sigma = 897.955\ 1\varepsilon^{0.025\ 25\ln\dot{\varepsilon} + \frac{441.973\ 6}{T} - 0.687\ 99}\dot{\varepsilon}^{\frac{-94.051\ 04}{T} + 0.284\ 11}\exp(-0.005\ 842T + 0.910\ 001\ 12\varepsilon)$
Al-1Si	$\sigma = 1\ 024.616\ 9\varepsilon^{0.025\ 05\ln\dot{\varepsilon} + \frac{443.863\ 3}{T} - 0.691\ 47}\dot{\varepsilon}^{\frac{-54.662\ 75}{T} + 0.228\ 99}\exp(-0.005\ 769T + 0.680\ 591\ 16\varepsilon)$
Al-1Si-0.5Cu	$\sigma = 1\ 190.944\ 2\varepsilon^{0.017\ 34\ln\dot{\varepsilon} + \frac{283.492\ 4}{T} - 0.481\ 9}\dot{\varepsilon}^{\frac{-47.960\ 26}{T} + 0.209\ 9}\exp(-0.005\ 716T + 0.527\ 580\ 72\varepsilon)$

2.3.3 Modified Zerilli-Armstrong (m-ZA) Model

For face-centered cubic (FCC) materials, the expression of the original ZA model is as follows:

$$\sigma = C_0 + C_2\varepsilon^n[\exp(-C_3T + C_4T\ln\dot{\varepsilon}^*)] \tag{8}$$

where C_0, C_2, C_3 and C_4 are the material constants, $\dot{\varepsilon}^* = \dot{\varepsilon}/\dot{\varepsilon}_0$ with $\dot{\varepsilon}$ and $\dot{\varepsilon}_0$ are strain rate and reference strain rate, respectively.

To improve the prediction accuracy of the original ZA model, Samantaray et al.[22] proposed an m-ZA model, which was also adopted by Li et al.[13] and Paturi et al.[19] to study the constitutive behavior of 7050 and AA7075-T6 aluminum alloys. The expression of the model is as follows:

$$\sigma = (C_1 + C_2\varepsilon^n)\exp[(-C_3 + C_4\varepsilon)T^* + (C_5 + C_6T^*)\ln\dot{\varepsilon}^*] \tag{9}$$

where $T^* = (T - T_{\mathrm{ref}})$ with T and T_{ref} are current temperature and reference temperature, respectively.

The methods and steps for determining the material constants in m-ZA model are shown in Fig 8. The constitutive equations of Al-0.5Cu, Al-1Si and Al-1Si-0.5Cu alloys obtained by the m-ZA model are shown in Table 3.

2.4 Comparison of Constitutive Models

The flow stress predicted by strain-compensated Arrhenius-type model is compared with the experimental results as shown in Fig 9. It can be seen that the strain-compensated Arrhenius-type model has a good prediction effect on all three alloys. The prediction deviation from low to high are Al-0.5Cu, Al-1Si-0.5Cu and Al-1Si. The different deviations are mainly related to the hot deformation behavior of the alloy and the modeling process of the Arrhenius-type model. By comparing the experimental true stress-strain curves of the three alloys, it can be seen that the dynamic softening degree of Al-0.5Cu alloy is lower than that of the other two alloys. Specifically, the stress-strain curves of Al-0.5Cu alloy under various test conditions are continuously rising without obvious platform. The Arrhenius-type model has a good prediction effect on strain hardening and dynamic softening stages of the alloy in the hot deformation process, but in some transition deformation stages, such as the coexistence of work hardening (WH) and dynamic recovery (DRV), its prediction effect is often not ideal. The main reason for this is that the material constants of the Arrhenius-type model are averaged under various strain rates during modeling. Therefore, the influence of strain rates on material parameters is ignored, which will lead to different prediction effects of the Arrhenius-type model under different strain rates[23].

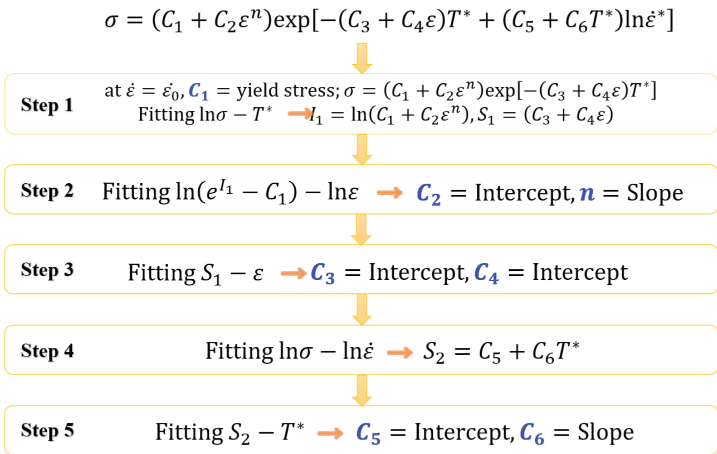

Fig 8 Methods and steps for determining material constants of m-ZA model

Table 3 Constitutive equations determined by the m-ZA model

Alloys	Constitutive equations
Al-0.5Cu	$\sigma=\left(25.74+26.131\ 2\varepsilon^{0.756\ 15}\right)\exp[(-0.005+0.002\ 75\varepsilon)T^*+(0.093\ 82+0.000\ 24T^*)\ln\dot{\varepsilon}^*]$
Al-1Si	$\sigma=\left(23.74+22.772\ 4\varepsilon^{0.531\ 66}\right)\exp[(-0.004\ 89+0.002\ 39\varepsilon)T^*+(0.114\ 75+0.000\ 16T^*)\ln\dot{\varepsilon}^*]$
Al-1Si-0.5Cu	$\sigma=\left(38+15.289\varepsilon^{0.640\ 03}\right)\exp[(-0.004\ 91+0.001\ 87\varepsilon)T^*+(0.112\ 25+0.000\ 125T^*)\ln\dot{\varepsilon}^*]$

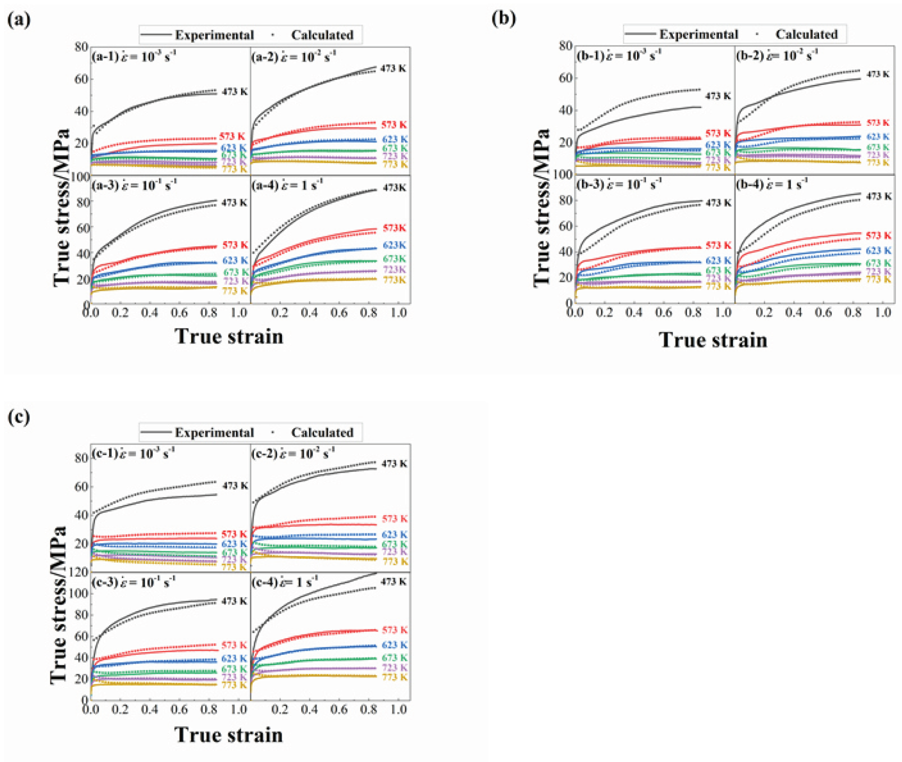

Fig 9 Comparison between experimental and predicted flow stress by strain-compensated Arrhenius-type model for (a) Al-0.5Cu; (b) Al-1Si and (c) Al-1Si-0.5Cu alloy systems

Fig 10 shows the comparison between the flow stress predicted by m-FB model and the experimental results. It can be seen that for the three aluminum alloys studied, the predicted results of the m-FB model have a large deviation from the experimental flow stress. The flow stress curve calculated by m-FB model tends to be linear, which cannot accurately describe the dynamic softening stage of the alloy. Relatively speaking, m-FB model can better describe the stress-strain curve at high temperatures (673~773 K), but at low temperatures (473~623 K), the description of flow stress is unsatisfactory.

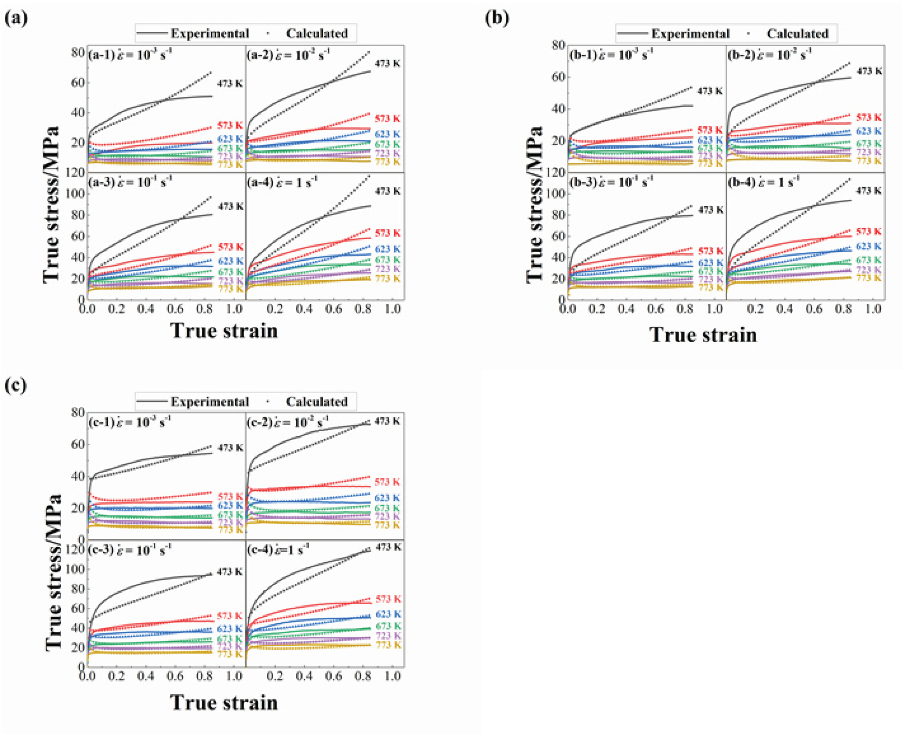

Fig 10 Comparison between experimental and predicted flow stress by m-FB model for (a) Al-0.5Cu; (b) Al-1Si and (c) Al-1Si-0.5Cu alloy systems

The comparison between the predicted results of m-ZA model and the experimental results is shown in Fig 11. It can be seen that the overall trend of the curve predicted by m-ZA model is also linear, but its prediction accuracy is obviously better than that of m-FB model. It also can be seen that the m-ZA model can accurately describe the dynamic softening behavior of Al-xSi-yCu alloy, but the prediction at the coexistence stage of work hardening and dynamic recovery is insufficient. The main reason is that the influence of strain on material parameters is not considered in the m-ZA model, which reduces the prediction accuracy of the model at small strain[24]. Comparing the prediction results of three alloys, it can be seen that the prediction effect of m-ZA model on Al-1Si alloy is better than that of the other two alloys. In addition, at each strain rate, the prediction effect of m-ZA model at lower temperature (473 K) is not ideal.

As can be seen from Figs 9~11, the flow stress predicted by the three constitutive models can track the experimental results in the entire domain. Inevitably, there are some deviations between the predicted flow stress and the experimental results. On the one hand, these deviations vary with the difference of test conditions and constitutive models; And on the other hand, these deviations fluctuate with the change of alloy system. To quantitatively compare the three constitutive models, two standard statistical parameters (correlation coefficient (R) and average absolute relative error (AARE)) are employed to quantify the predictability of the constitutive equations. The definitions of R and AARE are as follows:

$$R=\frac{\sum_{i=1}^{N}\left(E_i-\bar{E}\right)\left(P_i-\bar{P}\right)}{\sqrt{\sum_{i=1}^{N}\left(E_i-\bar{E}\right)^2\sum_{i=1}^{N}\left(P_i-\bar{P}\right)^2}} \tag{10}$$

$$\mathrm{AARE}(\%)=\frac{1}{N}\sum_{i=1}^{N}\left|\frac{E_i-P_i}{E_i}\right|\times 100 \tag{11}$$

where E is the experimental finding and P is the predicted value obtained from the constitutive equation. $\bar{E}$ and $\bar{P}$ are the mean values of E and P, respectively. N is the total number of data points employed in the investigation.

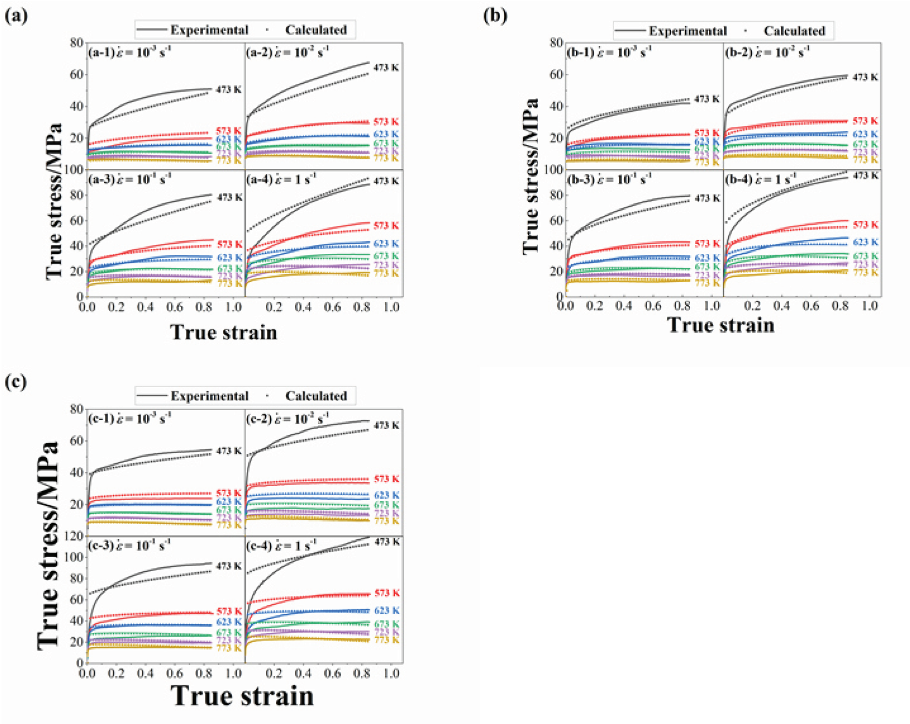

Fig 11 Comparison between experimental and predicted flow stress by m-ZA model for (a) Al-0.5Cu; (b) Al-1Si and (c) Al-1Si-0.5Cu alloy systems

The values of R^2 and AARE are summarized in Table 4. As can be seen from the table, in general, strain-compensated Arrhenius-type model has the highest prediction accuracy, followed by m-ZA model, while the result of m-FB model is less satisfactory. Fig 12 shows the correlation between experimental data and predicted flow stress of strain-compensated Arrhenius-type model, m-FB model and m-ZA model. The divergence between the predicted stress and the true stress measured experimentally can be seen from the figures. As mentioned previously, the reasons for the deviation between the prediction results and the experimental results are inseparable from the modeling process and parameter settings of the model.

Table 4 Statistical results of predictability of the three constitutive models

Constitutive models	Al-0.5Cu		Al-1Si		Al-1Si-0.5Cu	
	R^2	AARE/%	R^2	AARE/%	R^2	AARE/%
strain-compensated Arrhenius-type	0.992 6	5.51	0.970 0	8.74	0.983 4	7.98
m-FB	0.937 9	12.15	0.941 0	11.16	0.979 6	8.07
m-ZA	0.971 1	7.88	0.984 3	7.60	0.974 7	8.00

The perfect predictability of Arrhenius-type model is attributed to its hyperbolic sine function equation. The Zener-Hollomon parameter (Z) in the equation couples the influence of temperature and strain rate on deformation behavior, as well as the strain compensation for the parameters of its model equation. The high-order polynomial fitting of parameters promotes the high prediction accuracy of Arrhenius-type model in various strain ranges. It should be noted that compared with the other two constitutive models, the Arrhenius-type model has more material constants, and the determination of these material constants is more complex. According to relevant references[1,25−26], constitutive models with long calculation time and more material constants tend to have higher accuracy.

The m-ZA model considers two coupling effects through material constants[13]: 1) The deformation-heating effect couples the strain and temperature, which is represented by material constants C_3 and C_4. 2) The viscosity effect reflects the coupling between temperature and strain rate, and is represented by the material constants C_5 and C_6. In addition, the material constant C_1 is the yield stress under the reference condition, which means that the reference temperature and the reference strain rate affect the prediction accuracy of m-ZA model to some extent. Thus, it can be seen that the effects of strain, strain rate and temperature on the flow stress are not independent in both the Arrhenius-type model and m-ZA model, which makes the prediction accuracy of these two models acceptable in each alloy system.

Like the Arrhenius-type model, the m-FB model, which is also a phenomenological model, has only five material constants and is easy to calculate. Unfortunately, the prediction results of m-FB model are less satisfactory. Compared with the other two models, the prediction deviation of m-FB model is larger, and this deviation increases with the increase of stress, indicating that m-FB model is not the best choice to predict the flow behavior of Al-*x*Si-*y*Cu alloys. Although the coupling effect between parameters is also considered in the m-FB model, for example, the strain hardening coefficient (n) couples the relationship between temperature and strain rate, and the strain-rate sensitivity index (m) is expressed as a function of temperature, the interaction between various factors is not sufficiently considered.

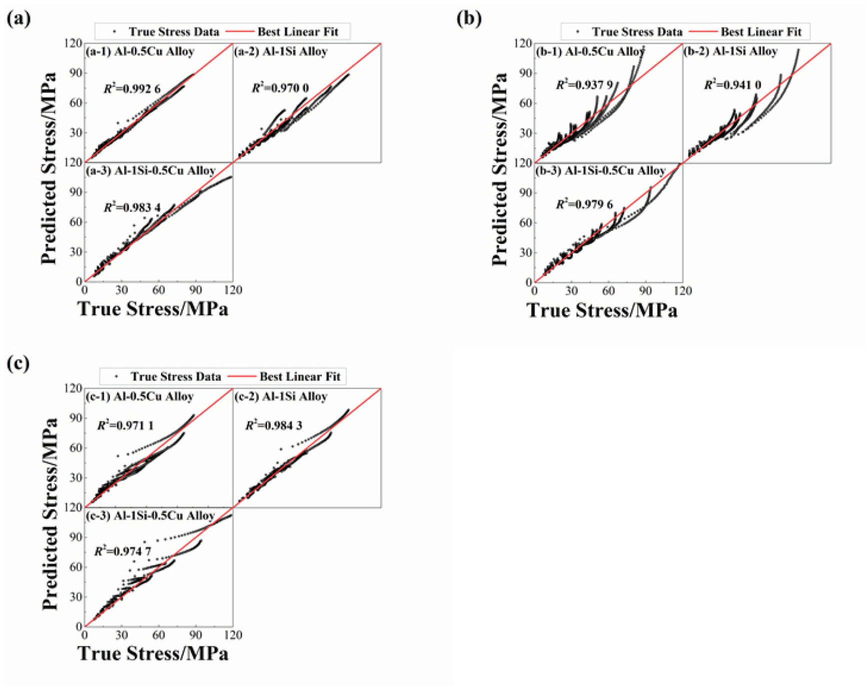

Fig 12 Correlation between experimental data and predicted flow stress of (a) strain-compensated Arrhenius-type model; (b) m-FB model and (c) m-ZA model

According to the calculation results, it can be found that the R^2 value of the constitutive model established in different alloy systems fluctuates between 0.937 9 and 0.992 6. To ensure the prediction accuracy of the constitutive model, this study assumes that when $R^2 \geq 0.985\ 0$, the flow stress calculated from the constitutive model can meet the requirements of the subsequent finite element model. To achieve this goal, the temperature and strain ranges in which the three constitutive models can meet the prediction accuracy are recalculated, that is, the application domains of these three constitutive models in different alloy systems.

The calculated application domains are summarized in Table 5. It can be seen that all the three constitutive models can meet the requirements of $R^2 \geq 0.985\ 0$ after modifying the temperature and strain ranges. However, the applicable temperature and strain ranges are different. To select the best constitutive model for each alloy system more conveniently, a group of figures is drawn based on the data in Table 5, as shown in Fig 13. It can be seen that for Al-0.5Cu alloy, the Arrhenius-type constitutive model has high prediction accuracy in the entire temperature and strain range, so it can be considered as the best constitutive model of Al-0.5Cu. Under high strain ($\varepsilon \geq 0.30$), m-ZA model can also meet the application requirements. However, the m-FB model can meet the application requirements only at high temperatures (673 K$\leq T \leq$773 K) and medium strains ($0.12 \leq \varepsilon \leq 0.48$). Different from Al-0.5Cu alloy, for Al-1Si alloy, m-ZA model is more suitable as the best constitutive model because of its wider application domain. The Arrhenius-type model also can be used as its constitutive model, but its accuracy will decline at low temperature ($T <$573 K) and low strain ($\varepsilon <0.20$). Similar to the case of Al-0.5Cu alloy, m-FB model can meet the application requirements only at high temperature (673 K$\leq T \leq$773 K) and medium strain ($0.20 \leq \varepsilon \leq 0.56$). For Al-1Si-0.5Cu alloy, both the Arrhenius-type model and m-ZA model meet the requirements of prediction accuracy. However, under the condition of low strain ($\varepsilon <0.10$), i.e. the initial stage of plastic deformation, the prediction accuracy of these two constitutive models is relatively low. Meanwhile, the m-FB model is still only applicable to high temperature (673 K$\leq T \leq$773 K) and medium strain ($0.10 \leq \varepsilon \leq 0.60$) conditions.

Table 5 Application domains of the studied constitutive models

Alloys	Constitutive models	R^2	AARE/%	Application domains	
				Temperature/K	Strain
Al-0.5Cu	Arrhenius-type	0.992 6	5.51	473~773	0.02~0.84
	m-ZA	0.985 4	6.08	473~773	0.30~0.84
	m-FB	0.985 1	8.50	673~773	0.12~0.48
Al-1Si	Arrhenius-type	0.985 5	6.78	573~773	0.20~0.84
	m-ZA	0.986 9	7.38	473~773	0.04~0.84
	m-FB	0.985 1	7.38	673~773	0.20~0.56
Al-1Si-0.5Cu	Arrhenius-type	0.985 2	7.58	473~773	0.04~0.84
	m-ZA	0.986 0	7.05	473~773	0.10~0.84
	m-FB	0.989 7	7.37	673~773	0.10~0.60

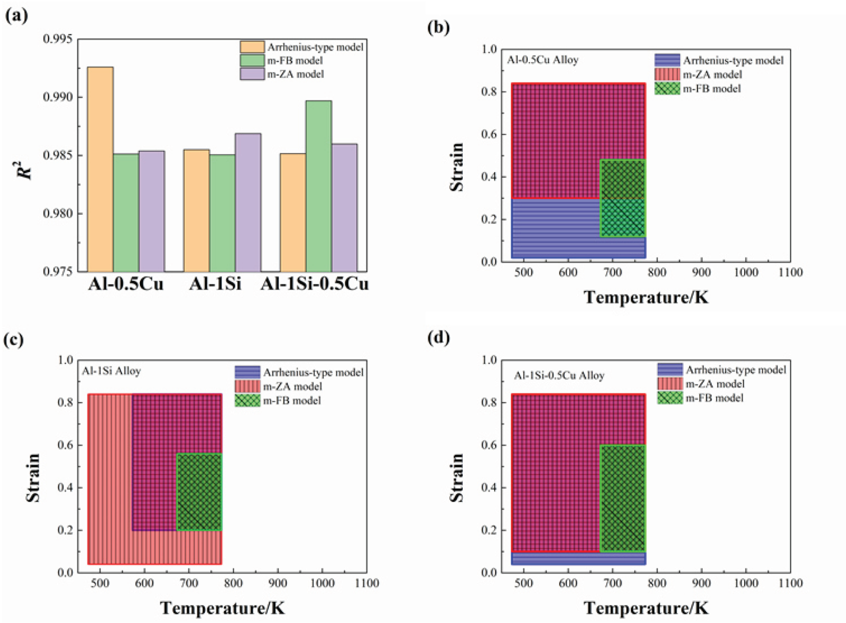

Fig 13 Comparison of prediction accuracy and application domain of the three constitutive models, (a) R^2; (b~d) application domain in Al-0.5Cu, Al-1Si and Al-1Si-0.5Cu alloy systems

Based on the above results, it can be found that the most suitable constitutive models for Al-0.5Cu, Al-1Si and Al-1Si-0.5Cu alloys selected from the three constitutive models are different. The reason can be attributed to two main aspects: Firstly, different alloys have different strengthening and deformation mechanisms, resulting in differences in work hardening, dynamic softening, dynamic recovery and dynamic recrystallization during hot deformation. Secondly, although the prediction accuracy of the strain-compensated Arrhenius-type model, m-FB model and m-ZA model is improved compared with the original models, these modified models cannot fully describe the various complex phenomena that may occur during the hot deformation process. The three constitutive models involved in the study have satisfactory prediction ability under certain conditions, and have practical application value. However, the application range of these three models is different. The common feature of these three models is that their prediction accuracy is not ideal at the initial stage of plastic deformation. Because at this stage, the material is in an unstable state, which is easily affected by various factors. For instance, the existence of temperature gradient will increase the stress (or strain) gradient, and the internal microstructure defects may affect the thermal deformation mechanism of the material at local locations. In general, both Arrhenius-type model and m-ZA model have good prediction accuracy in a wide range, while m-FB model has certain practicability only under high temperature and medium strain conditions.

3 Conclusions

Based on the characteristics of strain compensation Arrhenius-type model, m-FB model, and m-ZA model, as well as the prediction accuracy of flow stress, the most suitable constitutive models for Al-0.5Cu, Al-1Si, and Al-1Si-0.5Cu alloys were evaluated and selected. The conclusions drawn from the investigation are as follows:

1) The strain-compensated Arrhenius-type model couples the effects of temperature and strain rate on hot deformation by introducing Zener-Hollomon parameter (Z), and establishes a relationship between material constants and strain by high-order polynomial fitting, which makes the model have a good prediction effect in the entire strain and temperature range.

2) The m-ZA model considers the deformation-heating effect and viscosity effect, and couples the strain and temperature as well as the temperature and strain rate, which greatly improves the prediction ability of the model.

3) The strain-compensated Arrhenius-type model and m-ZA model are considered to be the most suitable constitutive models for Al-0.5Cu alloy and Al-1Si alloy, respectively, and also can be applied to Al-1Si-0.5Cu alloy. However, the m-FB model can be applied to Al-0.5Cu, Al-1Si and Al-1Si-0.5Cu alloys only under high temperature and medium strain conditions. Because the m-FB model has only a few material constants, the prediction results in the entire domain are less satisfactory. Only under specific conditions can the model meet the accuracy requirements and be applied.

References:

[1] LIN Y C, CHEN X M. A critical review of experimental results and constitutive descriptions for metals and alloys in hot working[J]. Materials & Design, 2011, 32(4): 1733-1759.

[2] SELLARS C M, MCTEGART W J. On the mechanism of hot deformation[J]. Acta Metallurgica, 1966, 14(9): 1136-1138.

[3] RUDNYTSKYJ A, SIMON P, JECH M, et al. Constitutive modelling of the 6061 aluminium alloy under hot rolling conditions and large strain ranges[J]. Materials & Design, 2020, 190: 108568.

[4] CHAMANFAR A, ALAMOUDI M T, NANNINGA N E, et al. Analysis of flow stress and microstructure during hot compression of 6099 aluminum alloy (AA6099)[J]. Materials Science and Engineering A, 2019, 743: 684-696.

[5] WU B, LI M Q, MA D W. The flow behavior and constitutive equations in isothermal compression of 7050 aluminum alloy[J]. Materials Science and Engineering A, 2012, 542: 79-87.

[6] SHI L, YANG H, GUO L G, et al. Constitutive modeling of deformation in high temperature of a forging 6005A aluminum alloy[J]. Materials & Design, 2014, 54: 576-581.

[7] TSAO L C, HUANG Y T, FAN K H. Flow stress behavior of AZ61 magnesium alloy during hot compression deformation[J]. Materials & Design, 2014, 53: 865-869.

[8] HUA L, MENG F Z, SONG Y L, et al. A constitutive model of 6111-T4 aluminum alloy sheet based on the warm tensile test[J]. Journal of Materials Engineering and Performance, 2014, 23(3): 1107-1113.

[9] TSAO L C, WU H Y, LEONG J C, et al. Flow stress behavior of commercial pure titanium sheet during warm tensile deformation[J]. Materials & Design, 2012, 34: 179-184.

[10] HUANG X M, WANG B Y, ZANG Y, et al. Constitutive relationships of 21-4N heat-resistant steel for the hot forging process[J]. Journal of Materials Research and Technology, 2020, 9(6): 13575-13593.

[11] JIA W T, XU S, LE Q C, et al. Modified Fields-Backofen model for constitutive behavior of as-cast AZ31B magnesium alloy during hot deformation[J]. Materials & Design, 2016, 106: 120-132.

[12] ZERILLI F J, ARMSTRONG R W. Dislocation-mechanics-based constitutive relations for material dynamics calculations[J]. Journal of Applied Physics, 1987, 61(5): 1816-1825.

[13] LI J, LI F G, CAI J, et al. Comparative investigation on the modified Zerilli-Armstrong model and Arrhenius-type model to predict the elevated-temperature flow behaviour of 7050 aluminium alloy[J]. Computational Materials Science, 2013, 71: 56-65.

[14] MAHALLE G, KOTKUNDE N, GUPTA A K, et al. Microstructure characteristics and comparative analysis of constitutive models for flow stress prediction of inconel 718 alloy[J]. Journal of Materials Engineering and Performance, 2019, 28(6): 3320-3331.

[15] LIU Y, LI M, REN X W, et al. Flow stress prediction of Hastelloy C-276 alloy using modified Zerilli-Armstrong, Johnson-Cook and Arrhenius-type constitutive models[J]. Transactions of Nonferrous Metals Society of China, 2020, 30(11): 3031-3042.

[16] LEE W S, LIU C Y. The effects of temperature and strain rate on the dynamic flow behaviour of different steels[J]. Materials Science and Engineering A, 2006, 426(1/2): 101-113.

[17] MIRZAIE T, MIRZADEH H, CABRERA J M. A simple Zerilli-Armstrong constitutive equation for modeling and prediction of hot deformation flow stress of steels[J]. Mechanics of Materials, 2016, 94: 38-45.

[18] ZHANG H J, WEN W D, CUI H T, et al. A modified Zerilli-Armstrong model for alloy IC10 over a wide range of temperatures and strain rates[J]. Materials Science and Engineering A, 2009, 527(1/2): 328-333.

[19] PATURI U M R, NARALA S K R, PUNDIR R S. Constitutive flow stress formulation, model validation and FE cutting simulation for AA7075-T6 aluminum alloy[J]. Materials Science and Engineering A, 2014, 605: 176-185.

[20] ZHAN H Y, WANG G, KENT D, et al. Constitutive modelling of the flow behaviour of a β titanium alloy at high strain rates and elevated temperatures using the Johnson-Cook and modified Zerilli-Armstrong models[J]. Materials Science and Engineering A, 2014, 612: 71-79.

[21] MANDAL S, RAKESH V, SIVAPRASAD P V, et al. Constitutive equations to predict high temperature flow stress in a Ti-modified austenitic stainless steel[J]. Materials Science and Engineering A, 2009, 500(1/2): 114-121.

[22] SAMANTARAY D, MANDAL S, BORAH U, et al. A thermo-viscoplastic constitutive model to predict elevated-temperature flow behaviour in a titanium-modified austenitic stainless steel[J]. Materials Science and Engineering A, 2009, 526(1/2): 1-6.

(下转第 23 页)